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RESEARCH ARTICLE

Computation of the Boltzmann entropy of a landscape: a review and a generalization

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Abstract

Context A key goal of landscape ecology is to understand landscape ecological processes across space and through time, with reference to the central organizing principles of nature. Towards this goal, Boltzmann (or thermodynamic) entropy has been widely used in a conceptual way to link these processes to thermodynamic laws, but it has seldom been computed because of a lack of feasible methods since its formulation in 1872. This situation will probably change because such methods have been developed very recently.

Objectives To present a timely, comprehensive review and an analysis of such methods.

Methods A systematic survey of the efforts to compute the Boltzmann entropy of a landscape was performed. The consistency of different computational methods was investigated.

Results In the review, two classes of methods were identified. The methods were developed from distinct ideas, apply to different landscape models (landscape mosaics and gradients), and result in different Boltzmann entropies. Thus, a general method for both landscape models would be desirable for consistent thermodynamic interpretations. Towards this goal, an approach was suggested to extend the method for mosaics to gradients or vice versa. Possible strategies for both extensions were theoretically analyzed and experimentally tested. Problems of each extension were revealed.

Conclusions These recently developed methods can be regarded as first steps in the computation of Boltzmann entropy for landscapes. This computation still requires much attention. Future research is recommended to improve the computation and to apply Boltzmann entropy in the thermodynamic understanding of landscape dynamics.

Keywords Boltzmann entropy · Thermodynamic entropy · Computational method · Landscape mosaic · Landscape gradient

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Introduction

The primary goal of landscape ecology is to understand the interaction between landscapes and underlying ecological processes (Turner 1989; Gardner and O'Neill 1991; Gillson 2009; Wu 2013). In this effort, one fundamental issue is the characterization of landscapes (Vogt et al. 2007; Riitters et al. 2009; Costanza et al. 2011); thus, a series of metrics have been employed (e.g., Riitters et al. 1995; Wickham et al. 1997; McGarigal et al. 2009; O'Neill et al. 2010; McGarigal et al. 2012; Gustafson 2018; Kedron et al. in review; Nowosad and Stepinski in review). Among these metrics, entropy is a fundamentally important metric because of its thermodynamic basis, which allows attractive potential interpretations of landscape dynamics with reference to the central organizing principles of nature.

According to a recent comprehensive review (Vranken et al. 2015), entropy has been applied to spatial, temporal, and spatio-temporal dimensions of landscape ecological research, as interpretations of spatial heterogeneity, the unpredictability of pattern dynamics, and pattern scale dependence, respectively. However, the following three observations made by Vranken et al. (2015) soon received much attention (e.g., Cushman 2015, 2016; Gao et al. 2017; Moffitt 2017; Gao et al. 2018b; Wang and Zhao 2018; Cushman 2018a, b):

> "Thermodynamic interpretation of spatial heterogeneity is not considered relevant.
> Thermodynamic interpretation related to scale dependence is also questioned by complexity theory.
> Only unpredictability can be thermodynamically relevant if appropriate measurements are used to test it." (Vranken et al. 2015, p. 51).

The reason behind these observations is the fact that the entropy of a landscape is computed according to or based on a model originating in communications (Cushman 2015; Vranken et al. 2015), i.e., the entropy proposed by Shannon (1948) and hence termed Shannon entropy. Such an entropy was originally used to quantify the information content of a telegram message (Shannon and Weaver 1949; Gibson 2002) and hence also termed information entropy. Although many improvements have been made to Shannon entropy to extend its applications (e.g., Li and Huang 2002; Claramunt 2005, 2012; Gao et al. 2018a), there is still no confirmed relationship between Shannon entropy and thermodynamics.

Instead of Shannon entropy, what should be used is argued to be Boltzmann (1872) entropy, and calls have recently been made to revisit it (e.g., Vranken et al. 2015; Cushman 2015, 2016; Sugihakim and Alatas 2016; Liang et al. 2018). Boltzmann entropy was selected for two reasons. First, it is theoretically capable of characterizing both the compositional and configurational disorder of a system, whereas Shannon entropy is capable of only one at a time. In this sense, Boltzmann entropy is more suitable for landscape characterization, where both composition and configuration matter. Second, Boltzmann entropy is the thermodynamic entropy (Sears and Salinger 1975; Atkins 1994; Kaufman 2002; Dalarsson et al. 2011), which is a key to interpret landscape ecological processes based on thermodynamic insights. Actually, thermodynamic entropy has long been conceptually used in landscape ecology for this purpose, such as when an ecosystem is seen as a dissipative system (e.g., Wu and Loucks 1995; Pelorosso et al. 2017) and when energy flows are considered in studying landscape ecological processes (e.g., Naveh 1987; Chapman et al. 2015). However, no method was developed for computing this entropy of a landscape until the last three years.

This study aims to present a timely and comprehensive review of the efforts to compute the Boltzmann entropy of a landscape. An analysis of the generalization of different computational methods will be carried out both theoretically and experimentally.

Boltzmann entropy and landscape models

Boltzmann entropy: concepts and difficulties in its computation

Boltzmann entropy was proposed by and named after the Austrian physicist Ludwig Boltzmann (1872). It is a measure of the disorder of a thermodynamic system, expressed through two notions, namely macrostate and microstate. A macrostate is a macroscopic description of the conditions of a thermodynamic system, whereas a microstate is a description from a microscopic point of view. One macrostate may correspond to a number of possible microstates, where only one microstate is the thermodynamic system in question. The number of possible microstates (*W*) can be used to determine the value of Boltzmann entropy (*S*) with the following equation (i.e., Boltzmann equation):

$$S = k_B \log(W) \tag{1}$$

where k_B is the Boltzmann constant $(= 1.38 * 10^{-23}\mathrm{J/K})$.

This equation is hard to solve because of two difficulties. One difficulty is the definition of the macrostate of a system, and the other is the determination of the number of possible microstates. As noted by Bailey (2009, p. 151), "researchers may not be certain how to specify and measure the macrostate/microstate relations." These two difficulties hold for landscapes. As observed by Vranken et al. (2015, p. 61), "no (thermodynamic) entropy quantification methods have been proposed" for landscapes. As a result, the use of Boltzmann entropy has long been limited to a conceptual level in landscape ecology.

Two models for representing a landscape: mosaic and gradient

In landscape ecology, there are two models for representing a landscape, namely mosaic (Forman 1995) and gradient (McGarigal and Cushman 2005).

By a mosaic (or patch-mosaic) model, a landscape is represented as a mosaic of discrete patches (i.e., a group of adjacent cells of the same cover class; Pearson and Gardner 1997) and thus is referred to as a landscape mosaic. A typical example is a land cover/land use map. This model is the traditional paradigm of landscape ecological research.

By a gradient model, a landscape is represented as a grid of quantitative attributes, which led to the term landscape gradient. Examples include a digital elevation model (DEM) and a remote sensing image. This model is regarded as being more general than the mosaic model because it subsumes the latter as a special case (McGarigal and Cushman 2005).

Because of these two models, efforts to develop computational methods for Boltzmann entropy have been made in two directions, i.e., computations with landscape mosaics and with landscape gradients. The remainder of this review will be organized accordingly.

Boltzmann entropy for landscape mosaics

Initial idea: to count the exact number of microstates in a macrostate by the total edge

The pioneering work on computing the Boltzmann entropy (also referred to as configurational entropy by Cushman) of a landscape mosaic was carried out by Cushman (2016), who provided insightful thoughts on the computation through two experiments. The first experiment is a thought experiment, and the other is a computer simulation.

In the thought experiment, the macrostate of a landscape mosaic was specified using three variables of cell composition, namely the dimensions (*D*) of a landscape mosaic, the number (*N*) of the cover classes of cells, and the proportions (*P*) of each cover class. For example, the macrostate specified for the landscape mosaic shown in Fig. 1 is as follows: $D = 3\times3$, $N = 2$, and $P = \{4/9, 5/9\}$. Then, the microstates were specified as cell configurations. In this case, one can imagine that any two landscape mosaics with the same cell composition (i.e., *D*, *N*, and *P*) will have the same number of possible microstates and thus the same Boltzmann entropy. In other words, the Boltzmann entropy computed in this way has the same performance as Shannon entropy: both are irrelevant to cell configuration.

In the second experiment, all the possible landscape mosaics were first simulated under the constraints that $D = 3 \times 3$, $N = 2$, and $P = \{4/9, 5/9\}$. This means that all the simulated landscape mosaics (see some examples in Fig. 2) share the same cell composition and thus the same Shannon entropy. Then, the landscape metric total edge (*TE*) was computed with each simulated landscape mosaic, as shown in Fig. 3.

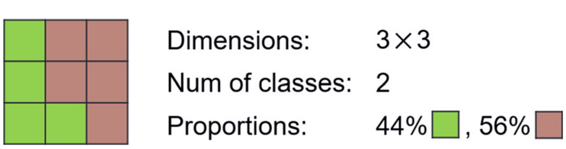


Fig. 1 Dimensions, number of cover classes, and proportions of each class of a landscape mosaic

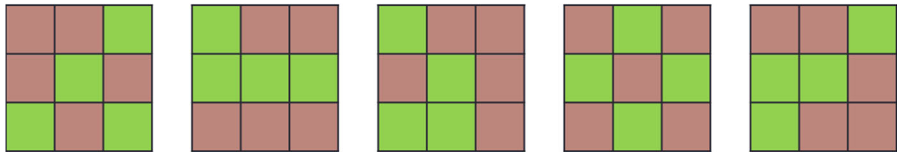


Fig. 2 Mosaics with the same dimensions, number of classes, and proportions as that in Fig. 1

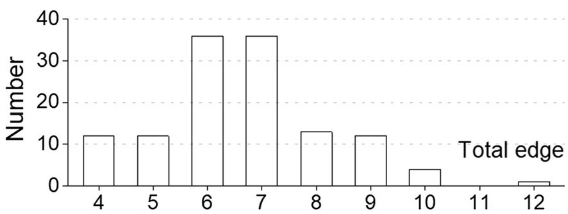


Fig. 3 Total edge and the corresponding number of simulated landscapes

It can be found from this figure that the number of simulated landscape mosaics changes with *TE*.

Based on the findings of the two experiments, Cushman (2016) developed the first-ever idea of computing the Boltzmann entropy of a landscape mosaic. That is, the macrostate was defined as the *TE* of a landscape mosaic, and the number of possible microstates was specified as the number of possible cell configurations that could be generated with the same cell composition (i.e., *D*, *N*, and *P*) and *TE* as the original, as shown in Fig. 4.

Implementation: relative entropy based on the estimated proportion of microstates

A key step in implementing the preceding idea is to find all the possible landscape mosaics that have the same *D*, *N*, and *P* as the original landscape mosaic. This task is hardly practical with a large *D* because of the considerable number of possibilities. For example, the number of possible landscape mosaics is as large as 1.03×10^{119}, even if *D* is only 20×20 ($N = 2$ and $P = \{50\%, 50\%\}$), and it increases dramatically with *D*. Therefore, this task has become the "fundamental challenge" (Cushman 2016, p. 486) in implementation.

To avoid this challenge, Cushman (2018a) proposed an alternative route with which to compute the Boltzmann entropy of a landscape mosaic. The route involves three steps.

1. Since it is impractical to find all the possible landscape mosaics (i.e., the population), he proposed generating a sample of the possible landscape mosaics instead. This generation was performed by randomizing the cell configuration of an original landscape mosaic a large number of times, for example, 100,000 as by Cushman (2018a).
2. Instead of counting the number (N_{TE}) of possible landscape mosaics with a given *TE* in the population, the proportion (P'_{TE}) of the possible landscape mosaics with the given *TE* in the sample was computed. P'_{TE} was regarded as an estimate of the proportion (P_{TE}) of such possible landscape mosaics in the population.

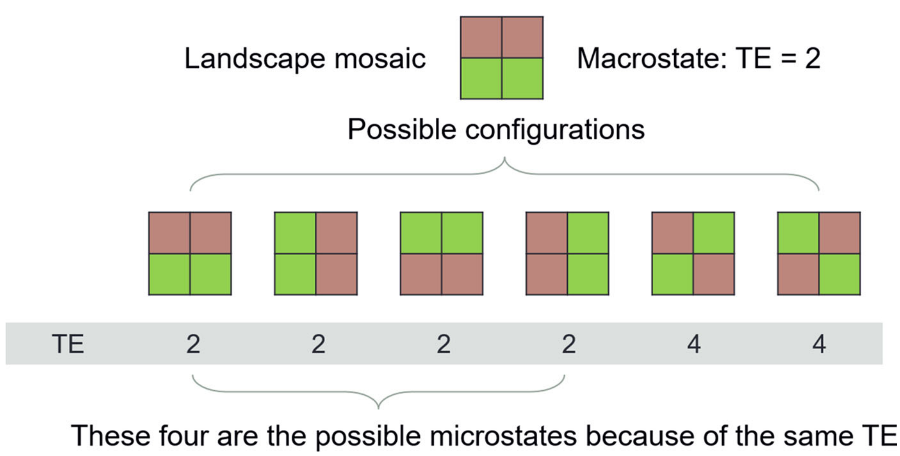


Fig. 4 Macrostate, possible cell configurations, and possible microstates of a landscape mosaic

3. Instead of N_{TE},P'_{TE} was substituted for the W in Eq. (1). The result, as shown in Eq. (2), is called relative entropy because it is relative to sample size.

$$S' = k_B \log(P'_{TE}) \tag{2}$$

It is worth noting that P'_{TE} may equal zero if the sample size is not large enough. Let us take the landscape mosaic in Fig. 5 as an example. It is a 16 × 16 multifractal map created using QRULE (Gardner 1999; Gardner and Urban 2007), and its *TE* equals 156. This original landscape mosaic was randomized 10,000 times in this study. However, none of the randomized landscape mosaic had a *TE* of 156, meaning that $P'_{TE} = 0$.

To solve this problem, Cushman (2018a) proposed to estimate P'_{TE} as follows:

$$\hat{P}'_{TE} = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(te-\mu)^2}{2\sigma^2}} \tag{3}$$

where μ and σ are the mean and the standard deviation of the (*TE*)s of all randomized landscape mosaics, respectively; and *te* is the *TE* of the original landscape mosaic. The justification for this estimation is his demonstration that (*TE*)s of all the randomized landscape mosaics for any landscape mosaic follow a normal distribution.

In summary, the alternative route to computing the Boltzmann entropy of a landscape mosaic is to compute the relative entropy by estimating the proportion of microstates based on the normal distribution of the (*TE*)s of randomized landscape mosaics, that is,

$$\hat{S}' = k_B \log\left(\frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(te-\mu)^2}{2\sigma^2}}\right) \tag{4}$$

Note that the determination of μ and σ still requires a large amount of computation, specifically, 100,000 (or even more) randomizations of an original landscape mosaic and the same number of computations of the landscape metric *TE*. Therefore, to improve efficiency, Cushman (2018a) suggested an estimation of μ and σ and even a prediction of $\hat{S}'$. These attempts have not been thoroughly evaluated and can be found in the Appendix.

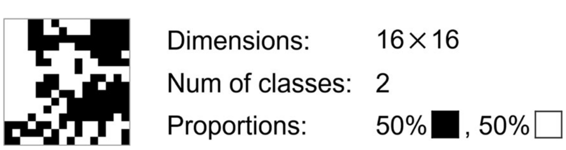


Fig. 5 Landscape mosaic created using the QRULE program

Boltzmann entropy for landscape gradients

The method for computing the Boltzmann entropy of a landscape gradient was originally proposed by Gao et al. (2017). It was then improved by Gao et al. (2018b) and Nowosad (2018).

Basic idea: macro- and microstate as different levels in a hierarchy

A landscape gradient can be transformed into a hierarchy that contains a series of landscape gradients with different levels of detail (Fig. 6). The relationship between two levels in such a hierarchy is similar to that between the macro- and microstate of a system. Inspired by such a line of thought, Gao et al. (2017) adopted a hierarchical perspective to define the macrostate of a landscape mosaic and to determine the number of possible microstates.

The idea for defining the macrostate is to select one level from the hierarchy of a landscape gradient as the macrostate. To set the criteria for this selection, Gao et al. (2017) revisited an example that is widely used in thermodynamics to illustrate Boltzmann entropy computation (e.g., Gould and Tobochnik 2010), namely a container filled with four gas molecules. By analyzing the successful definition of the macrostate in that example, Gao et al. (2017) established two rules for defining a good macrostate, as follows:

1. The macrostate should be general enough that it portrays the categorical states of the particles in a thermodynamic system.
2. The macrostate should be specific enough that it is capable of distinguishing two thermodynamic systems.

According to Rule 1, each hierarchical level can serve as the macrostate of a landscape gradient. If Rule 2 is also applied, the best candidate level will be the one that is most similar to the original landscape gradient. Therefore, the macrostate was defined as follows: an upscaled landscape gradient created by applying a

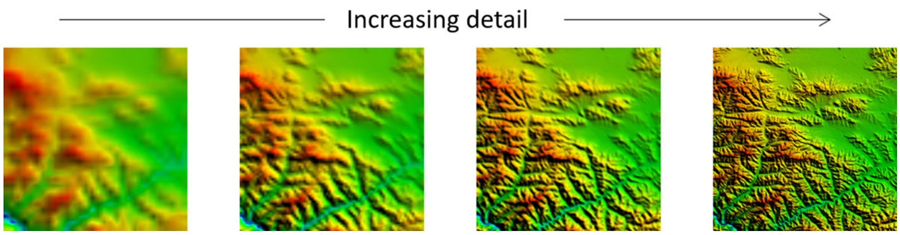


Fig. 6 A landscape represented with different levels of detail

2 × 2-cell moving average filter to the original landscape gradient (every four cells fall into the filter were referred to as original cells by Gao et al. 2018b).

The idea for determining the number of microstates is to count how many possibilities there are to downscale the macrostate to the original resolution. However, the possibilities appear endless, as shown in Fig. 7, so some constraints are needed. Thus, three principles were introduced for the downscaling, as follows:

1. Maximum preserved: a microstate has the same maximum as the macrostate.
2. Minimum preserved: a microstate has the same minimum as the macrostate.
3. Average (sum) preserved: a microstate has the same sum as the macrostate.

As a result, the number of possibilities became limited. This number varies with the landscape gradient, resulting in different Boltzmann entropies, as shown in Fig. 8.

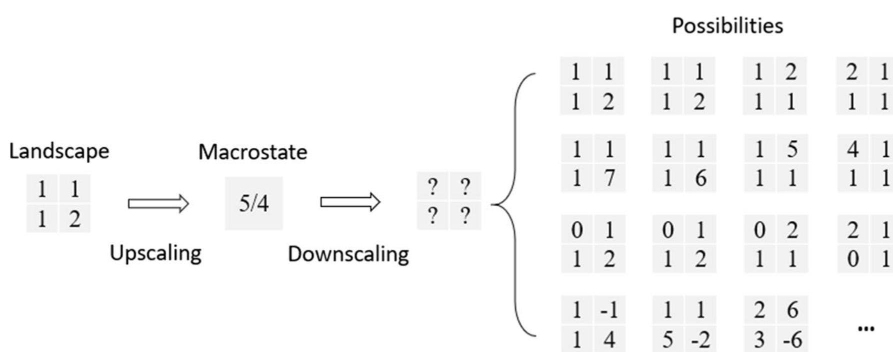
Careful readers may have noticed that all the input (original landscape gradient) and output (microstates) of Figs. 7 and 8 have integer values. This is actually an important assumption of Gao et al. (2017). In dealing with a landscape gradient of non-integer values, one should convert these values into integer ones. Such a conversion, however, could be an issue when the range of these non-integer values is small, e.g., 1.124–1.363.

Implementation: relative and absolute Boltzmann entropies

An immediate question is how to downscale the macrostate of a landscape gradient that has larger dimensions than those illustrated in Fig. 8 (i.e., has more than 2 × 2 cells). The answer depends on the understanding of a landscape gradient.

According to McGarigal and Cushman (2005), a landscape gradient is “a continuous surface or several surfaces corresponding to different environmental attributes” (p. 115). To model a landscape as a surface or several surfaces, three approaches are available (Li et al. 2004), namely point-based, triangle-based, and grid-based modeling. Gao et al. (2017) understood a landscape gradient to be a series of surfaces created using grid-based modeling. Accordingly, the downscaling of the macrostate of a large landscape gradient was performed as the downscaling (referred to as decomposition by Gao et al. 2018b) of every cell (i.e., aggregated cell) of the macrostate to four cells (decomposed cells). Since these decompositions are mutually independent, the number of microstates can be computed as the product of the numbers of the

Fig. 7 Endless possibilities when downscaling the macrostate of a landscape without constraints



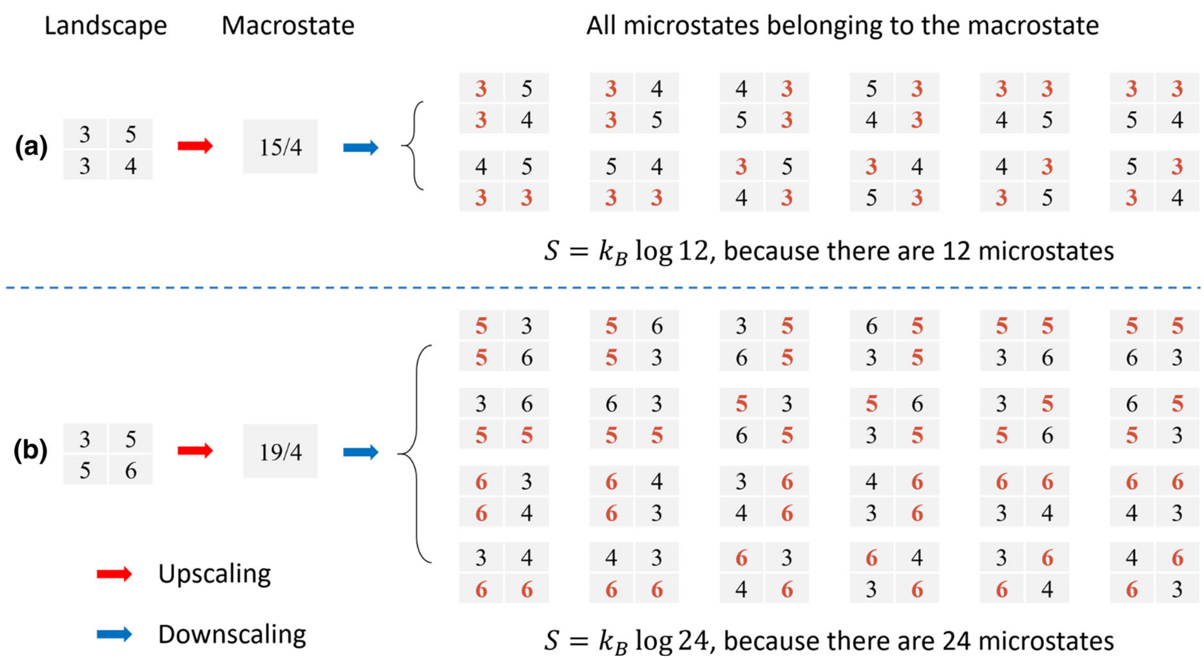


Fig. 8 Macrostates, microstates, and Boltzmann entropies (S) of two landscape gradients

possible outcomes of each decomposition. The resultant entropy with this number of microstates is referred to as relative (Boltzmann) entropy (S_R) because it characterizes the uncertainty of downscaling from the macrostate of a landscape gradient to its original resolution. In other words, this entropy is relative to a macrostate, which is only one level in the hierarchy of a landscape gradient.

One can imagine that a relative entropy can be computed for each hierarchical level. The sum of all these relative entropies is termed absolute (Boltzmann) entropy (S_A). It characterizes the uncertainty of downscaling from the most abstract level (i.e., a single cell) in the hierarchy of a landscape gradient to the most detailed level (i.e., the original landscape gradient), as previously illustrated by Gao et al. (2017) in their Fig. 9.

The relationship between relative and absolute entropies can be compared to that between relative and absolute heights. Only the comparison in absolute entropy between two landscape gradients of different dimensions seems meaningful because the absolute entropy is in reference to zero entropy (i.e., a single cell). According to the experiments by Gao et al. (2017), absolute entropy is capable of characterizing both the composition and configuration of a landscape in a sensitive manner.

Improvements: efficiency and border effect

There are two improvements to the computational method for a landscape gradient. One is to improve the efficiency (Gao et al. 2018b), and the other is to avoid the potential effect of the border of a landscape gradient (Nowosad 2018).

After implementing their basic idea, Gao et al. (2018b) found it inefficient in computing a Boltzmann entropy, especially its absolute value (i.e., S_A). This inefficiency is caused by so-called *decomposition* tasks, which are performed in a numerical way (i.e., by enumerating all possibilities) and, more seriously, are of an extremely large number when dealing with large landscape gradients (because each cell in a hierarchy is to be decomposed).

Two efforts were made towards the efficiency problem by Gao et al. (2018b). First, an analytical solution was developed to compute the number (i.e., M) of possible outcomes of a decomposition, as shown in Eq. (5). Second, a parallelization strategy was developed to perform the decomposition of the cells at different hierarchical levels in parallel.

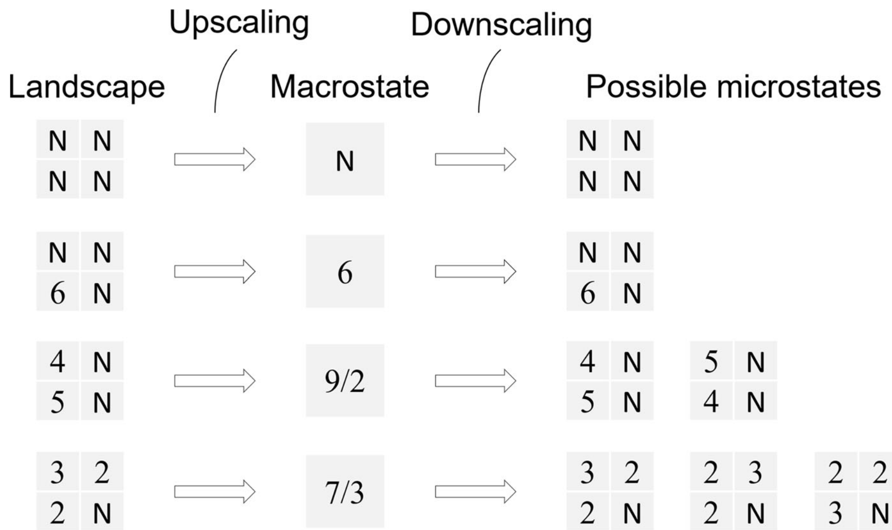


Fig. 9 Upscaling and downscaling with null-value cells (denoted by "N")

$$M = \begin{cases} 1 & d = 0,\ d_a = d_b,\ x_a = x_b \\ 6 & d = 0,\ d_a = d_b,\ x_a \neq x_b \\ 4 & d = 0,\ d_a \neq d_b,\ x_a = x_b \\ 12 & d = 0,\ d_a \neq d_b,\ x_a \neq x_b \\ 24(d-1)+18 & d \neq 0,\ d_a = d_b,\ x_a = x_b \\ 24(d-1)+30 & d \neq 0,\ d_a = d_b,\ x_a \neq x_b \\ 24(d-1)+24 & d \neq 0,\ d_a \neq d_b,\ x_a = x_b \\ 24(d-1)+36 & d \neq 0,\ d_a \neq d_b,\ x_a \neq x_b \end{cases}$$

where

$$\begin{cases} x_a = \dfrac{s - max - min}{2} \\ x_b = \dfrac{s - max - min}{2} \\ d_a = x_a - min \\ d_b = max - x_b \\ d = \mathrm{MIN}(d_a, d_b) \end{cases} \tag{5}$$

where *max*, *min*, and *s* are the maximum, minimum, and sum of the values of four original cells, respectively; $\lfloor\rfloor$ and $\lceil\rceil$ round a value down to and up to its nearest integer, respectively; and MIN() returns the minimum from a group of numbers.

The experimental results by Gao et al. (2018b) showed that with this parallel analytical method, the computation of Boltzmann entropy could be performed in near real-time on an ordinary computer. Related software tools have been made publicly available by Nowosad (2018) and Gao et al. (2018b).

The second improvement is to avoid the border effect of a landscape gradient. Sometimes the boundary of the landscape of interest is not regular, such as a map of census block groups (e.g., Buyantuyev and Wu 2010), the nighttime light distribution in a city (e.g., Wu et al. 2018), and the land cover of a mountain (e.g., Wickham et al. 2013). When such a landscape is modeled as a regular landscape gradient, some cells of the landscape gradient may have a null value, making the computational method of Boltzmann entropy inapplicable. To solve this problem, Nowosad (2018) specified how to perform upscaling and downscaling with null-value cells. The specification can be summarized as the following two rules:

In upscaling, the average is computed as the mean of non-null values, and
In downscaling, the number and positions of null-value cells are preserved.

Some applications of these rules are shown in Fig. 9.

Generalization of mosaic- and gradient-based entropies

We now have two classes of methods for computing the Boltzmann entropy of a landscape. One applies only to landscape mosaics, resulting in a mosaic-based

Boltzmann entropy. The other is only applicable to landscape gradients, leading to a gradient-based Boltzmann entropy. It would be desirable to have a general computational method that applies to both landscape mosaics and gradients. One approach towards this goal is to extend the existing methods, either from landscape mosaics to gradients or vice versa.

There are at least three possible strategies to extend the method for landscape mosaics to gradients. The first strategy is to incorporate a data preprocessing component into the method. When the input is a landscape gradient, its cells can be categorized into a small number of cover classes of interest before computing the Boltzmann entropy. However, the shortcoming of this strategy is that some information of the landscape will be lost after categorization. The second strategy is to simply treat gradient cells of different quantitative values as mosaic cells of different cover classes. This strategy seems promising, although it can result in numerous cover classes. For example, when dealing with a DEM of a mountain, there might be thousands of "cover classes" because the elevation may have a large range. The third possible strategy has been suggested by Cushman (2018a): instead of using the *TE* as a macrostate, one can employ the sum of the value differences between every cell and its eight neighbors when dealing with a landscape gradient. This strategy is theoretically attractive, but it would face the same challenge as using the *TE* as a macrostate: it would be impractical to count the exact number of possible microstates.

In this study, the second strategy was implemented to extend the method for landscape mosaics to gradients. Then, the extended method was tested using the same experimental data for testing the original method for landscape gradients (Gao et al. 2017), specifically, four pairs of DEMs in which the first is more disorderly than the second, as shown in Fig. 10. The $\hat{S}'$ of each DEM was computed with the extended method. In the computation of $\hat{S}'$, every DEM was randomized 100,000 times (which was used by Cushman 2018a). Since this number of randomizations may affect the value of $\hat{S}'$, we also set the number to 200,000 and 500,000 to investigate its effect. Therefore, for each DEM, we obtained three values of $\hat{S}'$. However, all three values of each DEM were $k_B \log 0$ (details in Table 1). This fact demonstrates that the extension strategy is not effective. In addition, it reveals that the number of randomizations has little

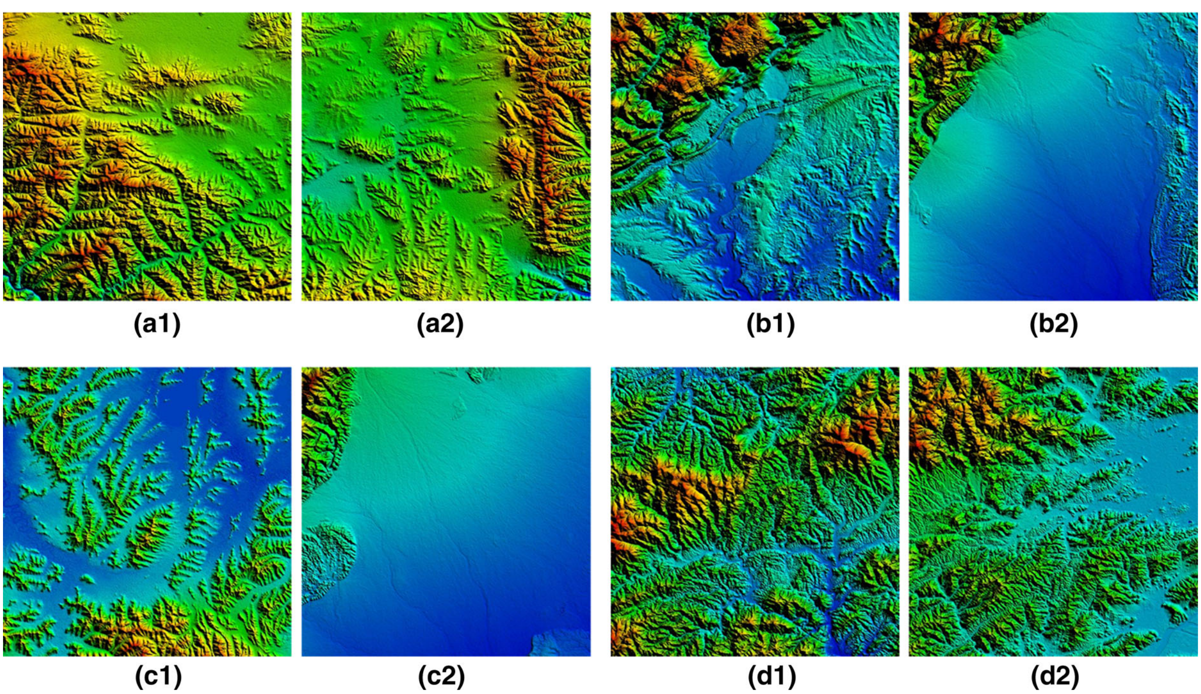


Fig. 10 Four pairs of digital elevations models (DEMs)

Table 1 Boltzmann entropies ($\tilde{S}'$, computed by extending the method for mosaic) of DEM in Fig. 10

DEM	Total edge	μ of Eq. (4)			σ of Eq. (4)			$\tilde{S}'$
		100 k	200 k	300 k	100 k	200 k	300 k	
a1	674,718	717,560	717,560	717,560	140	102	70	$k_B \log 0$
a2	666,204	717,084	717,084	717,084	166	121	83	$k_B \log 0$
b1	669,318	716,749	716,750	716,750	157	115	81	$k_B \log 0$
b2	571,666	715,529	715,530	715,530	458	327	211	$k_B \log 0$
c1	615,829	712,464	712,465	712,465	316	230	158	$k_B \log 0$
c2	537,000	715,361	715,362	715,362	567	403	259	$k_B \log 0$
d1	703,858	717,775	717,775	717,775	54	45	38	$k_B \log 0$
d2	669,871	716,516	716,517	716,517	155	115	81	$k_B \log 0$

"100 k", "200 k", and "300 k" denote that the number of randomizations is 100,000, 200,000, and 300,000, respectively. $\tilde{S}'$ here is found to be independent of this number

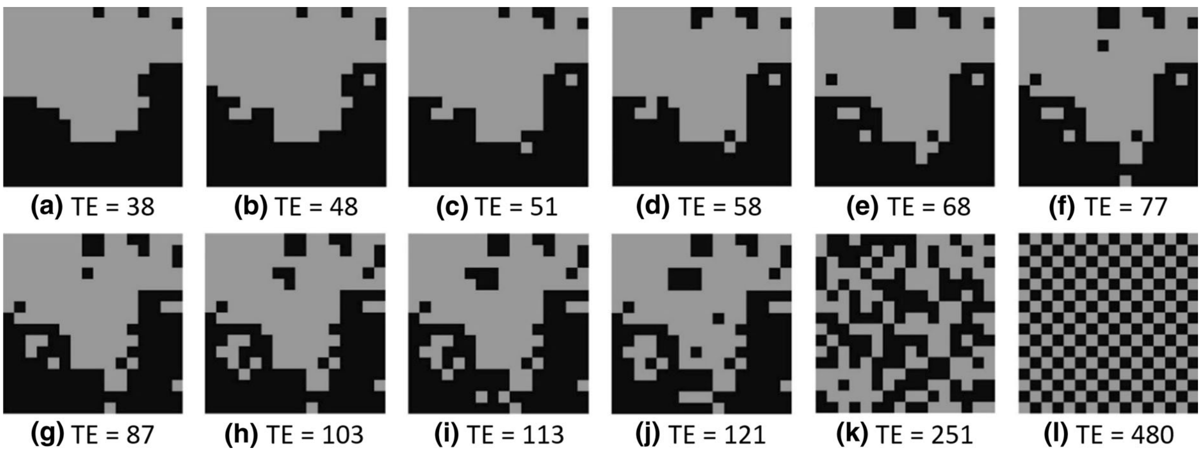


Fig. 11 Simulated landscape mosaics by Cushman (2018a) and their total edges (TEs)

effect. Since the strategy does not change the idea of the original method, the fact also suggests that the original method can be further improved when dealing with a landscape mosaic containing a very large number of cover classes.

Table 2 Relative Boltzmann entropies computed for the 12 mosaics in Fig. 11 by extending Gao et al. (2017) method and by Cushman (2018a), denoted by S_R and S', respectively

Mosaic	S_R	S'
a	52	− 167
b	67	− 153
c	71	− 148
d	80	− 137
e	97	− 123
f	106	− 112
g	118	− 96
h	138	− 77
i	150	− 67
j	165	− 47
k	316	− 4
l	403	− 250

The Boltzmann constant and logarithmic base were set to 1 and e, respectively

To extend the method for landscape gradients to mosaics, one may adopt the following strategy. For a landscape mosaic of only 2×2 cells, the macrostate can be defined as the number of cover classes and their proportions. Then, the number of microstates is computed as the number of possible spatial configurations given the same number of cover classes and their proportions. For the other landscape mosaics, the definition of macrostate and the determination of the number of microstates can be performed in a similar way as the original method for landscape gradients, namely by applying a 2×2 cell moving filter.

This strategy was implemented in this study and tested on landscape mosaics. The test data were the same as those used to test the original method for landscape mosaics (Cushman 2018a), specifically, the 12 landscape mosaics as shown in Fig. 11. The mosaics were ranked from the most aggregated (i.e., maximum homogeneity) to the most dispersed (i.e., maximum heterogeneity) in this study, according to their description by Cushman (2018a). The S_R of each mosaic is shown in Table 2, along with the Boltzmann entropy (S') computed by Cushman (2018a). From this table, the following observations can be made:

- Both S_R and S' characterized the disorder as increasing from Mosaic (a) to (k).
- The value of S_R indicated that Mosaic (l) was the most disorderly among the 12 mosaics, whereas that of S' showed that Mosaic (l) was the most orderly.

Discussion: checkerboard as most orderly or disorderly?

Should the checkerboard pattern (i.e., Mosaic l) be the most orderly (thus should have the lowest entropy) or disorderly (so have the highest entropy)? To try to answer this question, let us go back to thermodynamics.

The second law of thermodynamics tells us that "the entropy of a closed system increases continuously and irrevocably toward a maximum" (Huettner 1976, p. 102). A widely used illustration of this law is the mixing of two ideal gases. Such illustration can be found in most thermodynamic textbooks (e.g., Roy 2002; Gould and Tobochnik 2010). As shown in Fig. 12, there are two ideal gases initially separated by a partition in a closed container. Then, the partition is removed to allow their mixing. As a result, the entropy of the whole system increases until the two gases are fully mixed. Since the entropy of a closed system "can never decrease" (Bekenstein 2003, p. 61), entropy is sometimes called "an arrow of time" or "time's arrow" (Lebowitz 1993). It can be used to distinguish

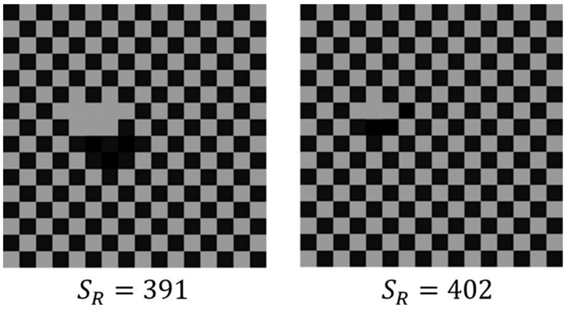


Fig. 13 Two mosaics that are similar to Fig. 11 (l) and their (S_R)s

the past from the future, following the principle that the past state has a lower entropy.

In a sense, the 12 mosaics of Fig. 11 can be regarded as different states of the mixing of two ideal gases (i.e., the black and the gray mosaic cells) in a closed container, as they have the same dimensions, number of classes, and proportions of classes. Since all 12 states are observable in the mixing (given enough time), how would one expect them to be ordered based on the time at which they can be observed? According to S', Mosaic (l) has a smaller entropy than (a), meaning that (l) comes earlier than (a). However, for a closed system where the entire process is spontaneous, such inference seems somewhat improbable. The dispersed gas molecules in a state of (l) have little chance of aggregating spontaneously to a state such as that of (a). By contrast, according to S_R, the time order of the 12 mosaics is from (a) to (l). In this case, (l) can be regarded as a theoretical state in which two gases are fully mixed. If the pattern of (l) is slightly changed to, say, the states shown in Fig. 13, S_R will be only slightly lower. This means that these states are very close to the theoretically fully mixed state.

Therefore, we argue that Mosaic (l) is not the most orderly among the 12 mosaics. In contrast, it is the most disorderly state in thermodynamic terms. In

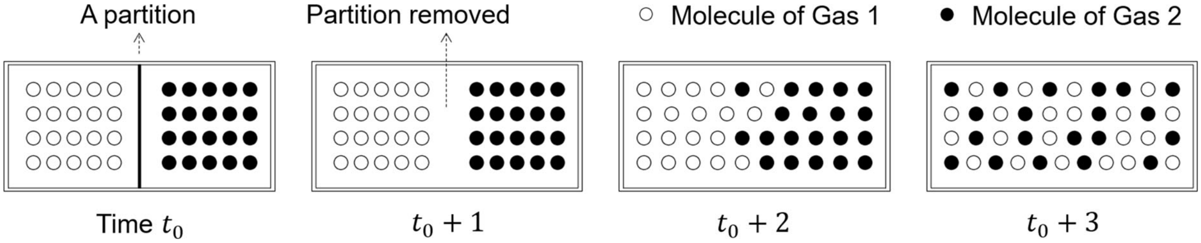


Fig. 12 Mixing of two ideal gases in a closed container

Fig. 14 Some quite different mosaics with the same total edge (TE = 38)

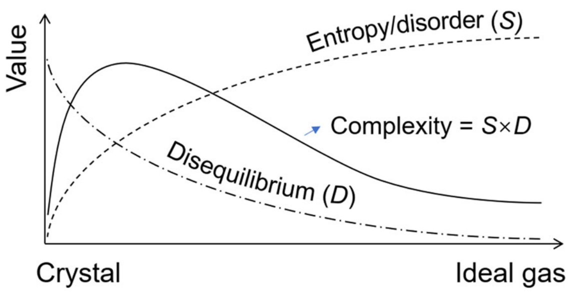


Fig. 15 Change of complexity from crystal to ideal gas. (modified from Lopez-Ruiz et al. 1995)

addition, Mosaic (k) should not have the highest entropy. It does, however, according to S'. This discrepancy might be caused by the macrostate definition of S', namely the TE. With this definition, two significantly different microstates are assigned to the same macrostate, as shown in Fig. 14. Since S' exhibits a parabolic trend when a thermodynamic system evolves from disequilibrium (e.g., the initial state of mixing) to equilibrium (e.g., the finial state of mixing), the characterization of S' might be the "complexity" of a thermodynamic system (Lopez-Ruiz et al. 1995), which has a behavior similar to that as shown in Fig. 15.

Concluding remarks

The potential fundamental importance of Boltzmann (or thermodynamic) entropy has been long and widely recognized in landscape ecology (e.g., Forman and Godron 1986; Naveh 1987; O'Neill et al. 1989; Wu and Loucks 1995; Zhang and Wu 2002; Cushman 2018b). Its use, however, remained at only a conceptual level. It was not until very recently that computational methods were developed for the Boltzmann entropy of a landscape.

This paper presented a timely and comprehensive review of these computational methods. It was found that two classes of methods have been developed: one applies only to landscape mosaics, and the other only to landscape gradients. However, a general method might be desirable for a model-independent thermodynamic understanding of landscape dynamics based on Boltzmann entropy. Accordingly, an analysis of possible generalizations of these methods was performed both theoretically and experimentally. This review concludes that the computation of the Boltzmann entropy of a landscape still requires attention and improvement, although some methods have been developed.

Future research is recommended in the following two areas. The first is to develop a general computational method of Boltzmann entropy. The second is to try to incorporate Boltzmann entropy into the thermodynamic understanding of landscape dynamics. Landscapes are widely understood as open systems or, more precisely, dissipative structures (Prigogine 1967; Prigogine et al. 1972) in terms of far-from-equilibrium thermodynamics. To quantify the entropy production resulting from irreversible processes inside the open system, Boltzmann entropy is a promising but poorly explored metric.

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