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Viewpoint: A correction to the entropy weight coefficient method by Shen et al. for accessing urban sustainability [Cities 42 (2015) 186–194]

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ABSTRACT

The assessment of urban sustainability is an essential step in planning, constructing, and maintaining sustainable cities, and thus it has become a research hotspot. In this context, Shen et al. (2015) accessed the sustainability of Jinan city (China) based on two indices calculated using an entropy weight coefficient method, and this journal published the results. However, we demonstrate that the calculated indices (and thus the assessment results) can never be obtained according to their "entropy weight coefficient method," by analyzing the mathematics behind the method and developing an approach to validating the results of the method. We find that this fact was caused by their mistakes in reporting the entropy weight coefficient method and, therefore, provide a necessary correction to these mistakes. We hope this correction is not too late, although the work has been cited 56 times (up to 5 May 2020).

1. Introduction

Sustainable development has been recognized as not only a common goal (World Commission on Environment and Development, 1987) but also an urgent need of humanity; thus, it appears to be the central theme of this century. In achieving sustainable development, urban sustainability plays a vital role and should be stressed for the following two reasons. First, we live in an age of "planetary urbanization" (Addie, Acuto, Ho, Cairns, & Tan, 2019; Antunes, March, & Connolly, 2020; Chen, Wen, & Li, 2017). It is projected that by 2050, two-thirds of the world's population will be living in cities (Kammen & Sunter, 2016). In some regions of the world (e.g., Northern America, Latin America, and the Caribbean), this rate had already surpassed 80% (United Nations, 2019). Second, cities are the centers for economic and social activities, and the economic and social dimensions are among the "three pillars" of sustainability (the other pillar is the environment, these three pillars are widely referred to as the triple bottom line).

In achieving urban sustainability, we need not only actions to change the current strategies of development (e.g., Aina, Wafer, Ahmed, & Alshuwaikhat, 2019; Chen et al., 2017; Gao, Cheng, & Song, 2019; Zhao, 2010) but also assessments of the progress (e.g., Huovila, Bosch, & Airaksinen, 2019; Zinatizadeh, Azmi, Monavari, & Sobhanardakani, 2017). In this context, Shen, Zhou, Skitmore, and Xia (2015) accessed and analyzed the sustainable development of a Chinese city, Jinan city of Shandong province, by applying a method called hybrid Entropy–McKinsey Matrix. This method is a combination of the entropy weight coefficient method and McKinsey Matrix. The entropy weight coefficient method was employed to calculate the weights of 52 indicators in accessing urban sustainability, such as violent crime rate per 10,000 population, gross domestic product, and industrial waste water emissions. Based on the 52 indicators and their weights, two indices were calculated, namely the development and coordination indices. A coordinate system was then defined: the *x*-axis and the *y*-axis range from zero to the theoretical maximum coordination index and the theoretical maximum development index, respectively. The coordinate plane was divided into 3 × 3 areas, forming a McKinsey Matrix. Based on this McKinsey Matrix of the development and coordination indices, further analysis was performed and a series of conclusions were drawn.

This work by Shen et al. (2015) is influential as it has been cited more than 50 times up to now, according to Google Scholar. We tried to repeat this work with a similar dataset, but completely different evaluation results of Jinan city were obtained. This fact made us revisit both the evaluation results reported by and the entropy weight coefficient method applied by Shen et al. (2015). Through a series of mathematical derivations, we found that there are serious mistakes in the "entropy weight coefficient method" applied by Shen et al. (2015).

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With that incorrect method, the evaluation results reported Shen et al. (2015) could never be obtained. This commentary aims to present our mathematical derivations and to provide a timely correction to the entropy weight coefficient method.

2. The "entropy weight coefficient method": a brief introduction

For the sake of completeness, we briefly introduce the "the entropy weight coefficient method" applied by Shen et al. (2015). Users of the "entropy weight coefficient method" are expected to have data on a number of indicators of urban sustainability measured for several consecutive years. Let us denote by n the total number of different indicators and m the total number of years. In this way, the users' data produce a matrix of $m \times n$ values. We summarize the "entropy weight coefficient method" as the following four steps:

1) Data normalization

Each value of the matrix is normalized to the range $[0,1]$. The normalization distinguishes between positive and negative indicators, as follows:

$$r_{ij} = \begin{cases} x_{ij}/\max\limits_{j}(x_{ij}) & \text{if } i \text{ is a positive indicator} \\ \min\limits_{j}(x_{ij})/x_{ij} & \text{if } i \text{ is a negative indicator} \end{cases} \tag{1}$$

where i and j denote the ith ($1 \leq i \leq n$) indicator and the jth ($1 \leq j \leq m$) year, respectively. x_{ij} denotes the value of the ith indicator for the jth year. $\max\limits_{j}(x_{ij})$ and $\min\limits_{j}(x_{ij})$ are the maximum and minimum values of the ith indicator among all years, respectively. A positive indicator is an indicator where a greater value means better sustainability; otherwise, the indicator is referred to as negative.

2) Entropy quantification

This step aims to quantify the information of each indicator with Shannon entropy (Shannon, 1948). The calculation of Shannon entropy requires a probability parameter. Usually, this probability parameter is determined as the frequency of occurrence of different values (Gao, Li, & Zhang, 2018). Here, the probability parameter is specified as the proportion (f_{ij}) of the value of an indicator for one year to the sum of the values of this indicator for all years. Also, a constant k was included in the equation of Shannon entropy, as follows:

$$\begin{cases} H_i = k\sum\limits_{j=1}^{m} f_{ij} \cdot \ln\dfrac{1}{f_{ij}} \\ f_{ij} = \dfrac{r_{ij}}{\sum_{j=1}^{m} r_{ij}} \\ k = 1/\ln m \end{cases}. \tag{2}$$

3) Weight determination

The weight of an indicator is negatively proportional to the Shannon entropy of the indicator. The equation for determining the weight (w_i) of the ith indicator is as follows:

$$w_i = (1 - H_i)/\sum_{i=1}^{n}(1 - H_i). \tag{3}$$

4) Index calculation

With the determined weights, two indices can be calculated. The first is a development index, which is a global assessment of sustainable development using all indicators:

$$F_j = \sum_{i=1}^{n} w_i \cdot f_{ij}. \tag{4}$$

Particularly, if the indicators can be classified into several categories, such as economic and environmental, a development index can be calculated for each category (φ), as follows:

$$F_{j(\varphi)} = \sum_{i\in\varphi} w_i \cdot f_{ij}. \tag{5}$$

The second is a coordination index, reflecting the difference among the degrees of sustainable development accessed with each category of indicators:

$$\begin{cases} C_j = 1 - S_j/\overline{F_j} \\ S_j = \sqrt{\dfrac{1}{u}\sum\limits_{v=1}^{u}(F_{j(\varphi_c)} - \overline{F_j})^2} \\ \overline{F_j} = \dfrac{1}{u}\sum\limits_{v=1}^{u} F_{j(\varphi_v)} \end{cases}, \tag{6}$$

where u is the total number of categories and φ_v denote the vth category.

3. A mathematical validation of the evaluation results by Shen et al. (2015)

In this section, we first develop a simple but effective approach for mathematically validating the results of the "entropy weight coefficient method." Then, we apply the validation approach to the evaluation results reported by Shen et al. (2015).

3.1. A validation approach for results of the "entropy weight coefficient method"

By analyzing the principles of the "entropy weight coefficient method," we proposed a mathematical approach for validating the resultant indices of the method. The approach is to check whether the following theorem holds or not: The sum of (F_j)s for all years equals one.

To prove this theorem, let us first express the sum of (F_j)s as follows:

$$\sum_{j=1}^{m} F_j = \sum_{j=1}^{m}\sum_{i=1}^{n} w_i \cdot f_{ij} = w_1\sum_{j=1}^{m} f_{1j} + w_2\sum_{j=1}^{m} f_{2j} + \cdots + w_n\sum_{j=1}^{m} f_{nj}. \tag{7}$$

Combining Eqs. (7) and (2), we found that

$$\sum_{j=1}^{m} F_j = w_1\frac{\sum_{j=1}^{m} r_{1j}}{\sum_{j=1}^{m} r_{1j}} + w_2\frac{\sum_{j=1}^{m} r_{2j}}{\sum_{j=1}^{m} r_{2j}} + \cdots + w_n\frac{\sum_{j=1}^{m} r_{nj}}{\sum_{j=1}^{m} r_{nj}} = w_1 + w_2 + \cdots + w_n. \tag{8}$$

Combining Eqs. (8) and (3), we found that

$$\sum_{j=1}^{m} F_j = \frac{1 - H_1}{\sum_{i=1}^{n}(1 - H_i)} + \frac{1 - H_2}{\sum_{i=1}^{n}(1 - H_i)} + \cdots + \frac{1 - H_n}{\sum_{i=1}^{n}(1 - H_i)} \equiv 1. \tag{9}$$

Therefore, the theorem has been proved.

3.2. Validating the results by Shen et al. (2015): inconsistency discovered

With the validation approach, we found an inconsistency between the method applied by and the evaluation results reported by Shen et al. (2015). In other words, the evaluation results of Jinan city can never be obtained using the "entropy weight coefficient method."

Specifically, the theorem does not hold with the reported evaluation results. As shown in Table 4 of Shen et al. (2015), the calculated (F_j)s for the years 2000–2011 were 0.273, 0.270, 0.314, 0.361, 0.394, 0.488, 0.663, 0.559, 0.570, 0.643, 0.699, and 0.888, respectively. The sum of these (F_j)s equals 6.1220, rather than one.

This finding demonstrates that either the "entropy weight coefficient method" was mistakenly reported or the evaluation results were incorrectly calculated. In the latter case, the conclusions drawn by Shen et al. (2015) may be totally wrong because their whole analysis was performed based on F_j (the index C_j was also calculated based on F_j).

4. Discussion on the problematic results: a correction to the method

According to a series of works using similar methods (e.g., Ding, Shao, Zhang, Xu, & Wu, 2016; Li, Zhao, & Suo, 2014; Wang et al., 2015), we tend to think the "entropy weight coefficient method" was mistakenly reported but correctly used by Shen et al. (2015). The development index calculated based on the entropy weight coefficient method should essentially be the weighted average of a group of indicators for sustainable development. According to this principle, we correct the "entropy weight coefficient method" reported by Shen et al. (2015) as follows: Eqs. (4) and (5) should be corrected as Eqs. (10) and (11), respectively.

$$F_j = \sum_{i=1}^{n} w_i \bullet r_{ij}, \tag{10}$$

$$F_{j(\varphi)} = \sum_{i \in \varphi} w_i \bullet r_{ij}. \tag{11}$$

Actually, the results reported by Shen et al. (2015) should have been calculated using this corrected entropy weight coefficient method. As noted in the preceding section, their calculated (F_j)s for the years 2000–2011 ranged from 0.273 to 0.888, and half (F_j)s were greater than 0.5. This fact suggests a hypothesis that those (F_j)s were not the weighted average of proportions (f_{ij}). If those (F_j)s were, most (F_j)s should have a value of around 0.08 (i.e., 1/12) because 12 years were involved. Therefore, we conclude that although we found mistakes in their method description, the results by Shen et al. (2015) were obtained using the correct entropy weight coefficient method and thus should be reliable.

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CRediT authorship contribution statement

Peichao Gao: Conceptualization, Writing - original draft, Writing - review & editing. **Xiangyu Wang:** Methodology. **Haoyu Wang:** Validation. **Changxiu Cheng:** Supervision.

Declaration of competing interest

None.

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