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A Pareto front-based approach for constructing composite index of sustainability without weights: A comparative study of implementations

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A R T I C L E I N F O

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A B S T R A C T

A composite index based on selected indicators is a widely used tool for guiding, monitoring, and evaluating a society's level of sustainability. However, determining the weight of each indicator is typically a methodologically problematic and highly controversial process. This paper proposes a weightless strategy for constructing composite sustainability indices based on the mathematical optimization concept of Pareto fronts. The core idea is to model each indicator as an individual objective and explore Pareto fronts within the resulting multi-objective solution space. In practice, a total of 24 typical implementations of the strategy were realized to represent four categories with varying parameter settings, i.e., straightforward/hierarchical implementations with/without avoiding basic indicator accuracy issues. Comparative experiments demonstrated that a hierarchical approach utilizing the goodness of variance fit-based (GVF = 0.80) natural breaks to nullify accuracy problems is the most effective implementation. To demonstrate its usefulness, the strategy implemented using this approach was applied to analyze the world's sustainability by revising the well-known sustainable society index. This study provides a novel paradigm of composite sustainability indices and represents the first assessment of world sustainability using multiple criteria (indicators) without weights.

1. Introduction

Every individual, group, nation, and country wants a sustainable planet (Li et al., 2020; Xu et al., 2020; Sun et al., 2023); however, achieving this goal has become more difficult than ever. According to the Sustainable Development Goals Report (United Nations, 2021), the COVID-19 pandemic has exacerbated previous delays in the development and implementation of sustainable environmental policies. More targeted action is needed, which in turn requires more effective tools to guide, monitor, and evaluate current strategies to enhance sustainability. Indeed, the fundamental importance and added benefits of better sustainability assessment tools have increasingly been recognized and highlighted. The potential benefits include increasing our understanding of sustainability, supporting government decision making, improving communication and participation, and resolving conflicts between stakeholders (Yigitcanlar et al., 2015).

Indicators are the most common tool used for sustainability assessment. Individual indicators characterize a single measurable or observable aspect of sustainability and therefore usually have a clear physical meaning. However, although a large set of individual indicators could serve as a comprehensive profile of sustainability (e.g., Wang et al., 2023), it is difficult to make regional or temporal comparisons (or benchmarks) using a complete set of indicators. A second type of indicator is a composite index, also known as a composite indicator (e.g., Nardo et al., 2005), an aggregated index (e.g., Buck et al., 2021), or aggregated indicator (e.g., Dhingra and Chattopadhyay, 2021). In this paper, we use the term "index" rather than "indicator" because the value of an index, which is a characterization of multiple aspects of sustainability, may not have a simple physical meaning. A composite index can be further subdivided into thematic and systemic indices (Kang, 2019). A thematic index combines a specified collection of individual indicators, whereas a systemic index is a combination of all available indicators; however, both indices capture complex concepts that are not visible and easily understandable from a single indicator (Shah et al., 2019). Compared with a set of individual indicators, a composite index not only characterizes multidimensional sustainability, but also

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simplifies regional and secular comparisons, integrations into decision making, and public communication.

Following the selection of a group of indicators, constructing a composite index entails three steps, namely, preprocessing, weighting, and aggregating. Weighting is the most important step and should be handled with great care. However, existing approaches to applying weights have been subject to severe criticism, as weighting is typically a methodologically problematic and highly controversial process. For example, the well-known sustainable society index (SSI; Wu et al., 2018; Witulski and Dias, 2020) was first calculated using a series of evenly weighted indicators because there is no scientific basis for different weighting of indicators (Van de Kerk and Manuel, 2008, p. 233). Later, due to a controversy over weighting, this index was no longer published as a single composite value (Saisana and Philippas, 2012).

To avoid the problems caused by weighting, this study proposes a weightless strategy for building a composite index based on Pareto front mathematical optimization. The strategy is applied herein to revise the SSI and analyze sustainability in 154 countries and regions. It represents a new paradigm for the construction of composite sustainability indices for the assessment of world sustainability.

2. A critical review on constructing composite index of sustainability

2.1. The state of the art

In the literature, there have been several comprehensive reviews on composite indices of sustainability (Kwatra et al., 2020; Pena et al., 2020, 2021). A consensus of these reviews is that weighting is inevitable in constructing a composite index of sustainability.

Weighting methods can be basically divided into subjective and objective categorizes, which are also referred to as explicit and implicit, respectively (Pena et al., 2020). Explicit weighting methods means that users directly assign accurate weights to all or some indicators, or provide useful information about the importance of indicators. According to degree of user involvement, such methods can be further divided into strong and weak explicit methods (Pena et al., 2020). By contrast, implicit weighting methods do not involve user's explicit information about the importance of indicators (Pena et al., 2021). Sometimes there is an additional category as a combination of these two categorizes.

According to their basis, weighting methods can also be divided into statistic-based and public/expert opinion-based (Gan et al., 2017). Statistic-based methods determine indicator weights based on the statistical characteristics of indicator values. By contrast, public/expert opinion-based methods derive indicator weights based on the opinions of participating public or experts. If the opinions come from experts, such methods are also called top-town methods and are more suitable for a global or national scale; If the opinions come from local stakeholder, such methods are also called bottom-up and are more suitable for a regional or urban scale (Kwatra et al., 2020). A simple case beyond these two categories is equal weighting, where all indicators are attached with the same importance.

2.2. Problems of subjective and objective weighting

Subjective weighting reflects the opinion of an individual or a group. Individual opinion-based weighting has several drawbacks: the resultant weights are not always accepted by another individual or the public; different individuals may produce diverse weights, thereby making a comparison between assessment results meaningless; and determining weights becomes exponentially complex if the number of indicators increases substantially. In similar fashion, group opinion-based weighting suffers from the same drawbacks, although there exists a special case: When the group is sufficiently large, equal weights are usually used to incorporate everyone's opinion. However, equal weights ignore the objective importance of different indicators, which can lead to misleading assessment results.

Objective weighting aims to avoid biases deriving from subjective opinion and is often implemented by analyzing the statistical characteristics (e.g., frequency distribution) of the values of each indicator. However, this method also has obvious drawbacks, most notably that the resulting weights can hardly be seen as a reflection of the objective importance of the various indicators because the weights vary with at least three factors. The first factor is the specific value of indicators, which changes if assessment units are different or are subject to secular variation. The second factor is the formation of value groups for analyzing statistical characteristics—e.g., groups of the values of a single indicator for all assessment units in a given time as compared with groups of the values of a single indicator obtained in different years for a single assessment unit. The third factor is the size of the value groups—e.g., values obtained over a specified number of years.

2.3. Illustrating the problem of objective weighting

The entropy weights (or entropy weight coefficient) method is overwhelmingly popular in constructing a composite index (Gao et al., 2015). It involves collecting the values of sustainability indicators over a period of several years (Shen et al., 2015). Let r_{ij} denote the normalized value of the i-th indicator ($1 \le i \le T_i$) of the j-th year ($1 \le j \le T_j$). These indicators are aggregated using a weighted-sum strategy whereby the weight of each indicator is determined using the information entropy of the indicator's values for different years.

Mathematically, the resultant composite index (F_j) of an assessment unit in the j-th year can be expressed as follows:

$$\begin{cases} F_j = \sum_{i=1}^{T_i} w_i \bullet r_{ij} \\ w_i = (1 - H_i) / \sum_{i=1}^{T_i} (1 - H_i) \end{cases} \tag{1}$$

where w_i is the weight determined for the i-th indicator. H_i represents the information entropy, which is calculated based on the indicator values in different years, that is,

$$\begin{cases} H_i = \dfrac{1}{\ln T_j} \sum_{j=1}^{T_j} f_{ij} \ln \dfrac{1}{f_{ij}} \\ f_{ij} = \dfrac{r_{ij}}{\sum_{j=1}^{T_j} r_{ij}} \end{cases} \tag{2}$$

Although a standard methodology, F_j has serious limitations that can result in misleading assessments. The weights calculated using information entropy can hardly be regarded as reflecting an indicator's objective importance. For example, suppose the sustainability of two regions is compared using three positive indicators—environmental, economic, and social. The data for the two regions are the normalized values (normalized range 0–10) of these three indicators over an eight-year period (2014–2021), as shown in Tables 1 and 2. These tables also show are the weights for each indicator (last column) and the composite index for each year (last row) calculated using the entropy weight coefficient method. The following problems can be seen in Tables 1 and 2:

1. Some indicators have a weight of 0, which misleadingly implies that they are not important at all. In fact, its weight is not a characterization of the objective importance of an indicator, but rather denotes temporal variation of its values.
2. To avoid dubious conclusions, such a characterization of temporal variation cannot be compared across regions. For example, the environmental indicator for Region 1 apparently has a greater weight than the social indicator because there is more temporal variation in the former indicator's values than in the latter; however, it has a smaller weight than the environmental indicator for Region 2, where the values remain more-or-less consistent over time.

Table 1
Values of the environmental, economic, and social indicators for Region 1 from 2014 to 2021, weights of each indicator (w_i), and composite indices (F_j) of each year.

	2014	2015	2016	2017	2018	2019	2020	2021	w_i
Environment	6	7	8	9	10	10	10	10	0.981
Economy	10	10	10	10	10	10	10	10	0
Society	9.8	9.7	9.6	9.5	9.4	9.3	9.2	9.1	0.019
F_j	6.072	7.051	8.030	9.010	9.989	9.987	9.985	9.983	–

Table 2
Values of the environmental, economic, and social indicators for Region 2 from 2014 to 2021, weights of each indicator (w_i), and composite indices (F_j) of each year.

	2014	2015	2016	2017	2018	2019	2020	2021	w_i
Environment	9.998	9.997	9.996	9.995	9.994	9.993	9.992	9.991	1
Economy	8	8	8	8	8	8	8	8	0
Society	9.1	9.1	9.1	9.1	9.1	9.1	9.1	9.1	0
F_j	9.998	9.997	9.996	9.995	9.994	9.993	9.992	9.991	–

3. A comparison between regions can also be misleading. Region 1 was less sustainable than Region 2 in 2019 (9.987 vs. 9.993) despite the fact that it was more sustainable in terms of all three single indicators (10 vs. 9.993, 10 vs. 8, and 9.3 vs. 9.1).

3. Materials and methods

3.1. Weightless strategy to constructing composite indices

As can be seen from the preceding sections, assigning weights to indicators to form a composite index is always methodologically problematic and highly controversial. The fundamental reason for this is that objective and comprehensive weights for a selection of indicators rarely exist. It is possible to develop a more objective weighting strategy; however, arguably, no objective strategy is capable of producing comprehensive weights because they are developed according to the limited features of the objective world (e.g., to some statistical characteristics of the values of each indicator).

This section presents an alternative direction. Specifically, the authors propose a novel strategy to construct a composite index using the indicators in a weightless strategy. The central idea is to employ the concept of Pareto front (also known as Pareto frontier or Pareto boundary). This principle is now the basis of multi-objective optimization (Deb et al., 2002), where its utility has been tested and proven. In the field of multi-objective optimization, many early algorithms were implemented using a scalarization-based solution, i.e., by maximizing or minimizing a proxy objective that is a combination of weighted objectives. Recent algorithms have employed a Pareto front-based solution to simultaneously maximize or minimize a range of given objectives (e.g., Song and Chen, 2018; Blank and Deb, 2020; Wang et al., 2021). The vast range of the latter algorithms demonstrates the validity and effectiveness of the concept of the Pareto front in avoiding the determination of weights for objectives.

The proposed strategy employs the Pareto front concept to avoid weight determination for indicators, using the order of Pareto fronts to compare different assessment units. To illustrate the core idea, assume that there are only two negative indicators in a sustainability assessment, as shown by the two axes in Fig. 1(a). A negative indicator means that an assessment unit is more sustainable than another, if it has a smaller value. In addition, the assumption is that these two indicators cannot be compared in terms of importance; therefore, units can only be assessed on an indicator-by-indicator basis. Each point in Fig. 1(a) denotes an assessment unit, and its position in the coordinate system is determined by indicator values. This figure shows that some assessment units are not less sustainable than the others in terms of every individual indicator, namely, Assessment Units 1–4. These units, which can be described as Pareto optimal, form a frontier called the Pareto front. The Pareto front can be iteratively formed if units on previous Pareto fronts are excluded from the assessment, hence dividing all units into a series of Pareto fronts. For example, three Pareto fronts are formed in the case of Fig. 1(b). Although these two indicators cannot be compared in terms of importance, it is safe to say that the assessment units on the i-th Pareto front are more sustainable than those on the $(i+1)$-th Pareto front, where $i = 1, 2, 3, \cdots$ As a result, one can construct a composite index based on an assessment unit belonging to a specific Pareto front. The most straightforward construction is to directly use the order of Pareto fronts as a composite index. In the case of Fig. 1(b), Assessment Units 1–4 have a composite index of 1; Assessment Units 5–6, 2; and Assessment Units 7–9, 3, meaning that Assessment Units 1–4 are more sustainable than 5–6 and far more sustainable than 7–9. Complex implementations of this strategy will be proposed in the following section.

The proposed strategy possesses some advantages over existing methods. First, this strategy can be applied not only to ratio indicators

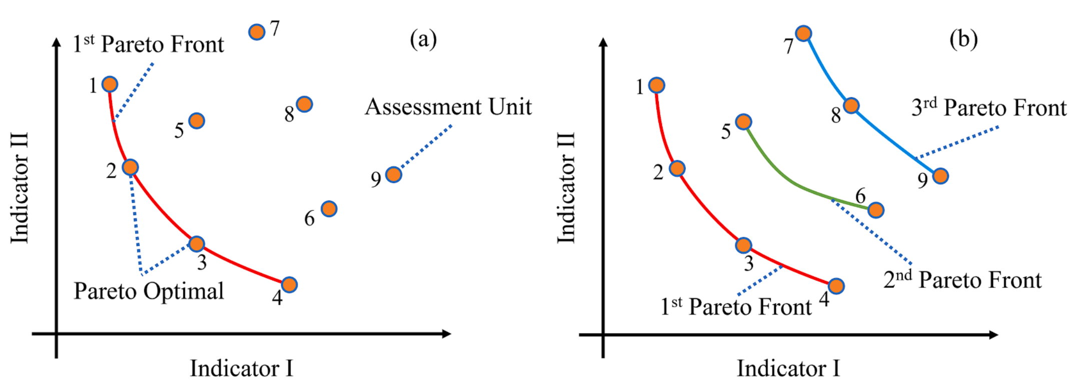

Fig. 1. Pareto fronts formed by assessment units. (a) Pareto front of all assessment units. (b) Pareto fronts formed by excluding the previous front iteratively.

(e.g., elevations) but also to interval (e.g., temperatures in Fahrenheit scale) and even ordinal indicators (e.g., high, medium, and low) because all these indicators can be used comparatively. In contrast, existing methods for constructing a composite index can only be securely applied to ratio indicators and—in some cases—interval indicators (if a geometric mean is not used in the aggregation step). Second, the proposed strategy is also applicable to correlated indicators, with the results unaffected by the correlation. In contrast, existing methods for constructing a composite index are unsuited for correlations between indicators because the effects of correlated indicators will be misleadingly highlighted in the resulting composite index. This is also why correlation reduction techniques have been incorporated into many construction techniques, such as factor analysis (e.g., Lv et al., 2021) and exploratory factor analysis (e.g., Mapar et al., 2020).

3.2. *Practical implementations of the weightless strategy*

The approach proposed in this study consists of two components. One is a strategy to construct a composite index without their weights, as described in the previous section. The other component is practical implementation of the strategy, which will be explained on in this section. To do so, the SSI will be used as illustrative data.

3.2.1. *Illustrative data: SSI*

The SSI is a country-level assessment of global sustainability in the form of composite indices (https://www.ssfindex.com/). Originally developed by the Sustainable Society Foundation, a non-profit organization based in the Netherlands, it has been improved by the Joint Research Center of the European Commission (Kaivo-oja et al., 2014) and is now operated by Technische Hochschule Köln (Kowalski and Veit, 2020). The first edition was calculated using 2006 data from 154 countries (or regions), covering more than 99% of the world's population. Subsequently, the SSI was updated every 2 years.

As a composite index, the SSI is based on 21 indicators. These indicators are grouped into seven intermediate categories, such as income distribution and biodiversity, and then into three dimensions, namely, human, environmental, and economic wellbeing (Fig. 2). In the official calculation of SSI, each indicator has a value (real number) normalized to the range from 0 (weakest) to 10 (strongest). These indicators are first aggregated to the categorical level using geometric means, and then intermediate categories are further aggregated to the dimensional level using the same method. Finally, dimensions are aggregated to be SSI using the arithmetic mean (Wu et al., 2018). Although there are disputes about the aggregation method (e.g., Gallego-Álvarez et al., 2015) and other similar, important indices (e.g., Kummu et al., 2018), the SSI is popular and was adopted for this study due to its simplicity, clear focus, and, especially, full disclosure of raw data.

3.2.2. *Straightforward and hierarchical implementations*

The most straightforward implementation was briefly introduced at the end of the strategy section. To calculate SSI, all 21 indicators are used to determine each possible Pareto front formed by included

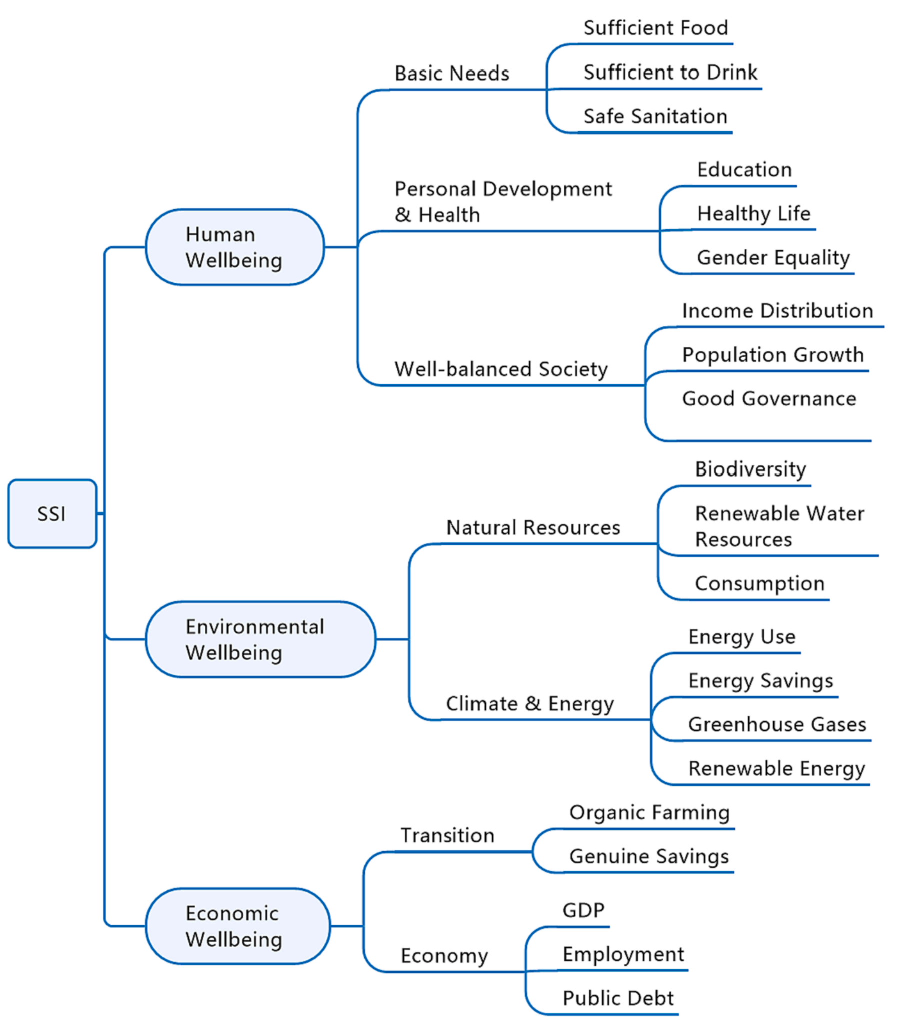

Fig. 2. Framework of the sustainable society index.

countries ($N = 154$). Let us assume that Country i (denoted as C_i, where $1 \leq i \leq N$) is determined to belong to the j-th Pareto front ($1 \leq j \leq M$). For easy reference, we call j the Pareto score of C_i and denote the determination of the Pareto score as a function $P(\bullet)$. Then, the composite index is straightforwardly calculated as $SSI_a = P(\Omega) = j$, where Ω is the collection of all indicator values for the country.

Since the indicators of SSI are organized in a hierarchy (Fig. 2), a straightforward implementation can be performed at each level. Specifically, this hierarchical implementation consists of three steps. First, the Pareto scores of all countries are calculated using each category of indicators (i.e., $\Omega_1, \Omega_2, \cdots, \Omega_7$). Thus, every country has seven Pareto scores (denoted as $P(\Omega_1), P(\Omega_2), \cdots, P(\Omega_7)$), each of which is an assessment of the country against a category. Then, new Pareto scores are calculated for every country using each dimension of the Pareto scores. As there are three dimensions, each country will now have three new Pareto scores. Finally, these new Pareto scores are used as input to calculate the composite index of each country (SSI_b), as follows:

$$SSI_b = P(P(P(\Omega_1), P(\Omega_2), P(\Omega_3)), P(P(\Omega_4), P(\Omega_5)), P(P(\Omega_6), P(\Omega_7))) \tag{3}$$

Note that there is a practical problem with both the straightforward and hierarchical implementations—to wit, the issue of indicator accuracy. It can be expected that the results of both implementations are highly sensitive to the accuracy of indicators, which is because the key of $P(\bullet)$ lies in the comparisons between two indicator values. An example is shown in Table 3, where an increased number of countries cannot be distinguished according to the value of biodiversity due to the decrease in accuracy. In this case, many countries will have the same or similar Pareto scores (as well as the values of the resultant composite index), thereby making the assessment less meaningful.

3.2.3. Implementations by avoiding accuracy issues

To avoid these accuracy problems, we developed two new implementations of the weightless strategy. Both implementations share the same core idea—to convert the values of each indicator, which are continuous, to discrete levels. For this conversion, two solutions are adopted, leading to two implementations by avoiding accuracy problems.

3.2.3.1. Solution 1: Conversion of values to levels by natural breaks. The first solution employed the "natural breaks" method for the conversion. This method is among the most widely used schemes to classify numerical values (Jenks, 1967; Yu et al., 2021; Gao et al., 2023). The core idea is to divide a group of numerical values into several sub-groups by applying an optimization algorithm, which aims to minimize the variance within each sub-group while also maximizing the variance between sub-groups. The classification results are usually consistent with human perception (hence the reason this method is called "natural") when the original values are visualized using a histogram, where the breaks fall into large gaps between two bins of the histogram.

A practical problem in employing the natural breaks method is determining the optimum number of breaks. This number is not computed through this method and must be subjectively determined by the user (Slocum et al., 2009). The experiment section of this paper performs tests with different numbers of breaks to elucidate this issue. The optimal number of breaks may be determined by controlling the goodness of variance fit (GVF) of the classification results, halting the optimization algorithm incorporated in the natural breaks method once the GVF has reached a given threshold. This determination strategy will also be tested.

3.2.3.2. Solution 2: Conversion of values to levels by head/tail breaks. The second solution uses head/tail breaks (Jiang, 2013; Jiang and Ma, 2018; Ma et al., 2020), which is an alternative to the natural breaks method. The advantage of this alternative method is that it can automatically determine the optimum number of breaks. It has been demonstrated that this alternative method "is more natural than the natural breaks in finding the groupings or hierarchy for data with a heavy-tailed distribution" (Jiang, 2013, p. 482), and it has been shown to be applicable to data without heavy tails (Gao et al., 2016; Zhang and Wu, 2020). The head/tail breaks method even serves as a fundamental basis to the so-called third-generation (or definition) of the fractals of geographical features (Gao et al., 2017; Ma and Jiang, 2018).

The core idea behind this method is rather simple. A group of values is first divided into two parts using their average; the part with fewer members is called the "head" and the other part the "tail." If the members of the head part are lower (in terms of values) than the average, the criterion for identifying a head part can be formulated as the values smaller than the average. Otherwise, the criterion is formulated as the values greater than the average. This average-based criterion is then applied iteratively to divide the head part, unless the members of a newly formed head part are not less than (in terms of the number of members) those of the corresponding tail.

In this study, we employed both the original version (as described above) and a revised version of the head/tail breaks method. We revised the method by replacing its average-based criterion with a minority-based criterion—i.e., a head part is further divided into a minority and a majority, with the minority designated as the new head. The revised version helps to create a more informative hierarchy of classes (or levels).

Table 3
Values of the biodiversity indicator displayed in different accuracy for some countries.

Country	Biodiversity			
	Accuracy 1	Accuracy 2	Accuracy 3	Accuracy 4
Albania	5.509	5.51	5.5	6
Algeria	6.595	6.6	6.6	7
Angola	4.101	4.1	4.1	4
Argentina	5.408	5.41	5.4	5
Armenia	7.755	7.76	7.8	8
Australia	7.335	7.34	7.3	7
Austria	7.918	7.92	7.9	8
Azerbaijan	7.291	7.29	7.3	7
Bangladesh	2.852	2.85	2.9	3
Belarus	5.899	5.9	5.9	6
Belgium	7.414	7.41	7.4	7
Benin	5.5	5.5	5.5	6
Bhutan	9.508	9.51	9.5	10
Bolivia	5.5	5.5	5.5	6
Bosnia-Herzegovina	3.175	3.17	3.2	3
Botswana	6.447	6.45	6.4	6

3.3. Experimental comparison of implementations

The different implementations of the proposed weightless strategy to constructing composite indices require testing to identify the most effective one. This need is met through comparative experiments as follows.

3.3.1. Dataset and settings

The dataset used in the comparative experiments comprises the values of 21 indicators for 154 countries (or regions) in 2016. The dataset can be obtained on the official website (https://ssi.wi.th-koeln.de/documents/data/2006–2016-countries.xlsx). As noted in the preceding section, each value is a real number in the theoretical range from 0 to 10. For the year 2016, not all the indicators have a real-world range from 0 to 10. For example, the indicator for healthy life has a value range from 4.1 to 9.2.

To perform a comprehensive evaluation, four groups were constructed and a total of 24 experiments conducted by combining the proposed weightless strategy and different implementations, as shown in Table 4. As can be seen, there are four accuracy settings, namely, original, 0.01, 0.1, and 1, which represent the following four numerical resolutions: infinite, 1,000, 100, and 10, respectively. The term "infinite" means that the value range between 0 and 10 allows for infinite assessment units with different scores when the accuracy is sufficiently high. Similarly, a numerical resolution of x (e.g., $x = 1000, 100, \text{or} 10$) indicates that the value range allows for up to x different scores. The natural breaks method was used by setting the number of levels ($L_{nb} = 4, 5, \text{or} 6$) and by setting the GVF ($= 0.50, 0.65, \text{or} 0.80$), referred to as L_{nb}-based and GVF-based natural breaks method, respectively. The head/tail breaks method was used by adopting its original average-based criterion and also by the proposed minority-based criterion for identifying a head part.

3.3.2. Criterion and measures for comparison

Comparing the results of different experiments requires a criterion of usability and specific measures. The International Organization for Standardization (ISO) defines usability as "the extent to which a product can be used by specified users to achieve specified goals" (ISO, 2008, p. 2) and evaluates usability against three criteria—effectiveness, efficiency, and satisfaction. The satisfaction criterion was not adopted in this paper because it is subjective in nature (Gao et al., 2021). The efficiency criterion was also excluded because none of the methods utilized in this study are computationally intensive. Thus, only the effectiveness criterion was applied and further specified to be discriminatory. In other words, an implementation is effective if it allows countries to be discriminated according to the resultant index.

To quantify the extent of discrimination, we proposed to use two measures. The first measure can be obtained in constructing the composite index—it is the number of Pareto fronts (N_{PF}) formed by all countries using the values of their composite index. The second measure is determined after calculating the composite index, which is the information entropy (i.e., Shannon entropy) of the index values of all countries. Information entropy originated in the field of information theory (Shannon, 1948) and has proven useful in myriad fields (please refer to Cushman, 2015; Gao and Li, 2019, for a parallel concept, i.e., thermodynamic entropy of Boltzmann entropy). It has served as a basis for digital communications (Shannon and Weaver, 1949) and is often applied to disciplines such as cartography (e.g., Li et al., 2021) and image processing (e.g., Zhang et al., 2020). In these fields, this entropy is interpreted as the information content of a dataset. Here it is used as a measure of discrimination by considering not only the number of different categories of values but also the proportion of each category, as follows:

$$H(X) = -\sum_{i=1}^{N_{PF}} P(X = X_i)\log_2 P(X = X_i) \tag{4}$$

where $P(X = X_i)$ is the proportion of countries on the i-th Pareto front. It is worth noting that both H and N_{PF} are positive measures—i.e., the greater their value, the more effective the construction approach.

Table 4
Implementations, parameters, and settings of the 24 experiments.

Group	Basic implementation	Parameter	Setting	Experiment
A	Straightforward	Accuracy	Original	A1
			0.01	A2
			0.1	A3
			1	A4
B	Hierarchical	Accuracy	Original	B1
			0.01	B2
			0.1	B3
			1	B4
C	Straightforward implementation by avoiding accuracy issues	Natural breaks	$L_{nb} = 4$	C1
			$L_{nb} = 5$	C2
			$L_{nb} = 6$	C3
			GVF = 0.50	C4
			GVF = 0.65	C5
			GVF = 0.80	C6
		Head/tail breaks	Average-based	C7
			Minority-based	C8
D	Hierarchical implementation by avoiding accuracy issues	Natural breaks	$L_{nb} = 4$	D1
			$L_{nb} = 5$	D2
			$L_{nb} = 6$	D3
			GVF = 0.50	D4
			GVF = 0.65	D5
			GVF = 0.80	D6
		Head/tail breaks	Average-based	D7
			Minority-based	D8

Note: L_{nb} = number of levels divided by natural breaks; GVF = goodness of variance fit.

4. Results and analysis

4.1. Comparison results

The results (N_{PF} and H) of all 24 experiments are shown in Table 5, including the ranks of the two measures—the higher the rank, the better is the performance. As shown in Table 5, the first group of experiments (A1–A4) have poor performance in terms of either N_{PF} or H. Specifically, Experiments A1–A3 result in an N_{PF} of 1 and a H of 0, thereby indicating a failure to discriminate any of the countries to be assessed in terms of sustainability. Therefore, the assessment does not provide any informational content. This type of performance was ranked last in all experiments. In contrast, Experiment A4 had slightly better results, with the ranks of both N_{PF} and H improved by 1. This group of experiments (A1–A4) demonstrated that straightforward implementation is not effective, and its effectiveness can only be slightly improved by changing the accuracy (i.e., numerical resolution) of the indicators.

The performance of the second group of experiments (B1–B4) was satisfactory and was affected by the accuracy of indicators. As shown in Table 5, all four experiments resulted in great (N_{PF})s, ranking from third to fourth in all 24 experiments. In terms of H, both Experiments B1 and B2 obtained a value greater than 3.5, ranking in the third and second places, respectively. However, along with the reduction of accuracy, Experiments B3 and B4 saw a sharp decrease in the rank of H (ranked 11th and 9th, respectively). Overall, these four experiments demonstrated that if hierarchical implementation is adopted, then users would be well advised to retain the original accuracy of indicators.

Table 5
Results of all the 24 experiments.

Experiment	N_{PF}	Rank of N_{PF}	H	Rank of H
A1	1	9	0	22
A2	1	9	0	22
A3	1	9	0	22
A4	2	8	0.174	21
B1	13	3	3.595	3
B2	13	3	3.608	2
B3	12	4	3.330	11
B4	12	4	3.367	9
C1	2	8	0.539	18
C2	3	7	0.516	19
C3	2	8	0.347	20
C4	5	5	2.118	14
C5	5	5	2.128	13
C6	5	5	1.419	17
C7	4	6	1.546	15
C8	4	6	1.518	16
D1	13	3	3.535	5
D2	13	3	3.423	7
D3	13	3	3.438	6
D4	12	4	3.243	12
D5	12	4	3.366	10
D6	15	1	3.693	1
D7	14	2	3.536	4
D8	13	3	3.400	8

The performance of the third group of experiments (C1–C8) was generally acceptable, displaying clear changes with the specific solution to avoid accuracy issues and the parameter setting. Specifically, obvious differences can be seen in both N_{PF} and H in Experiments C1–C3, C4–C6, and C7 and C8. Experiments C1–C3 resulted in the smallest N_{PF} and H, with the average N_{PF} and the average H being 2.333 and 0.467, respectively. In contrast, Experiments C4–C6 resulted in the highest N_{PF} (average 5) and H (average 1.888), followed by Experiments C7 and C8 with an average N_{PF} of 4 and an average H of 1.532. It should also be noted that Experiments C7 and C8 had similar results, suggesting that the implementation is barely impacted by the head/tail breaks method being average-based or minority-based. This group of experiments demonstrated that the GVF-based natural breaks method is the optimum solution for avoiding accuracy issues in the straightforward implementation, retaining effectiveness even with a small GVF (e.g., 0.50 in Experiment C4).

The best performance was observed with the fourth group of experiments (D1–D8). In terms of N_{PF}, Experiments D1–D8 had an average of 13.125, ranking first among all the four groups of experiments. The (N_{PF})s of every single experiment of this group ranked in the first four places among all 24 experiments. In terms of H, Experiments D1–D8 achieved an average of 3.454 and produced the highest value (3.693) among all the experiments. In terms of both N_{PF} and H, Experiment D6 performed the best, demonstrating that the most effective implementation is a hierarchical implementation using the GVF-based (GVF = 0.80) natural breaks method to nullify accuracy issues.

4.2. Case study: Spatio-temporal analysis of world sustainability

The experiments outlined in the previous section demonstrate that the proposed weightless strategy has the greatest efficacy with hierarchical implementation using the GVF-based (GVF = 0.80) natural breaks method to nullify accuracy issues. In this section, that implementation is applied to analyze world sustainability using the SSI dataset.

4.2.1. Spatial pattern of world sustainability in 2016

We calculated the composite indices of 154 countries (or regions) included in the SSI dataset of 2016, which is the most recent year covered by the index. The results are demonstrated in Fig. 3(a). Next, the results were analyzed at the continental and country scales.

The continents are ranked according to their overall performance, from the most sustainable to the least sustainable, as follows: Europe, North America, Oceania, Africa, Asia, and South America. As shown in Fig. 4, European countries obtained an average composite index of 3.72, whereas North America scored 4.57; Oceania 6.33; Africa 7.27; Asia 7.79; and South America 7.82. Measured by coordinated development, Europe was also the most sustainable continent, as the variance of the composite indices of all European countries was the lowest (5.7) of the six continents. The least coordinated continent was Africa, with a variance of 13.5 despite the fact that the value range of its composite indices (1–14) is similar to that of Asia (2–15).

At the country scale, these 154 countries (or regions) formed a total of 15 Pareto fronts (Fig. 3b–d). Worldwide, a total of 12 countries (or regions) obtained a composite index of 1 (Table 6). Half of these most

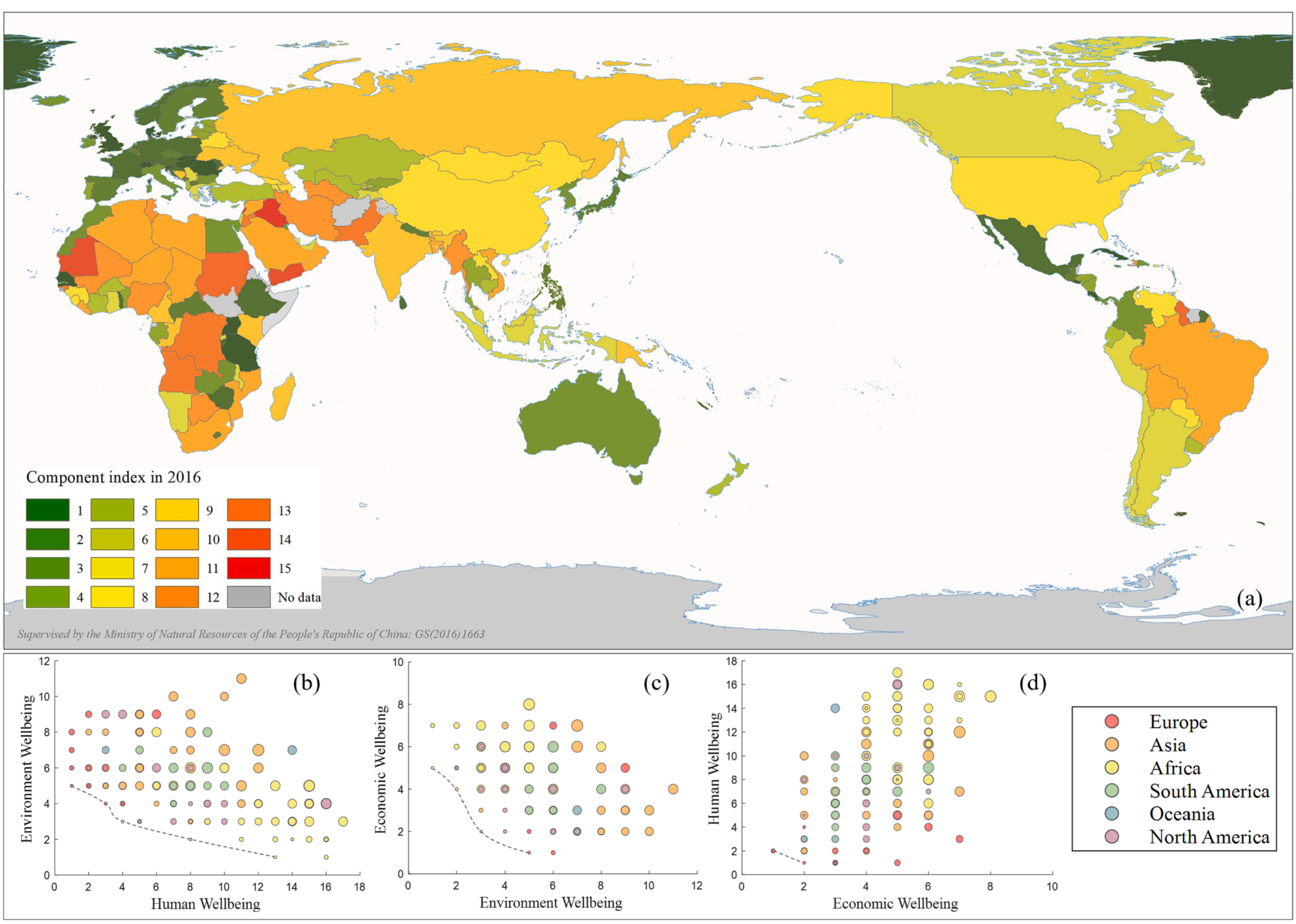

Fig. 3. Spatial pattern of world's sustainability in 2016 (a) and Pareto fronts formed along every two dimensions (b–d).

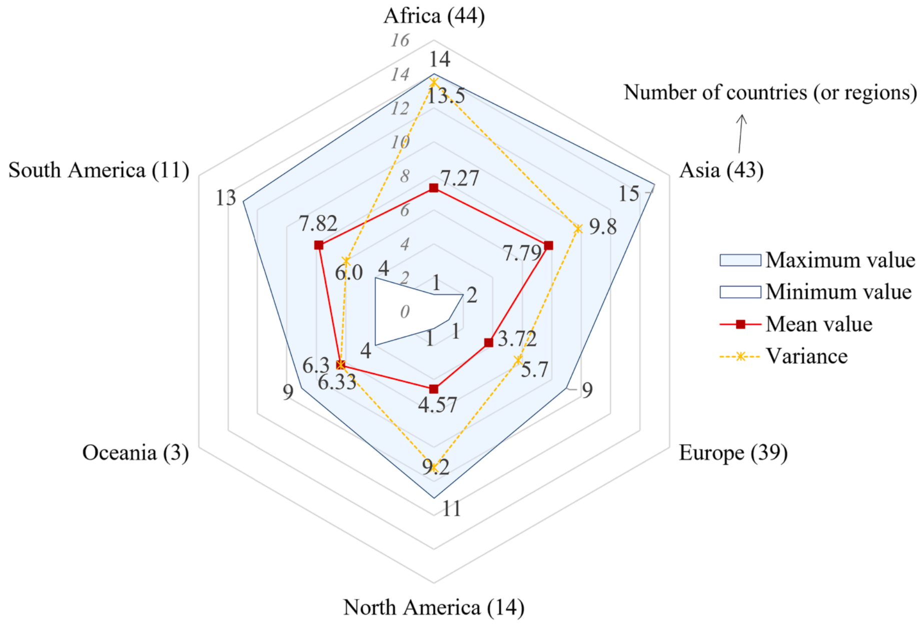

Fig. 4. Statistics of the composite indices by continent.

Table 6
Countries (or regions) that obtained a composite index of 1 in 2016 and their detailed performance in terms of human, environmental, and economic dimensions.

Continent	Country (region)	Composite index	Human	Environmental	Economic
Africa	Senegal	1	13	1	5
Africa	Tanzania	1	14	2	4
Africa	Uganda	1	14	2	4
Europe	Denmark	1	1	6	2
Europe	Hungary	1	3	4	3
Europe	Slovak Republic	1	3	4	3
Europe	Slovenia	1	1	5	3
Europe	Switzerland	1	2	5	1
Europe	United Kingdom	1	1	5	3
North America	Cuba	1	5	3	2
North America	Costa Rica	1	4	3	4
North America	El Salvador	1	8	2	5

sustainable assessment units were developed countries of Europe—Denmark, Hungary, Slovak Republic, Slovenia, Switzerland, and the United Kingdom. The other half were three developing countries in North America (i.e., Costa Rica, Cuba, and El Salvador) and the same number of countries from Africa (i.e., Senegal, Tanzania, and Uganda). As Table 6 indicates, the major reason for the excellent performance of the European countries is their overwhelming advantage against the human or economic dimension. The three North American countries had advantages in terms of both environmental and economic dimensions. For the three African countries, environmental advantages were sufficiently substantive to compensate for their disadvantages in the human wellbeing indicators.

The least sustainable countries in 2016 are shown in Table 7. Iraq had the highest composite index (15), followed by three countries with a composite index of 14 (i.e., Lebanon, Mauritania, and Yemen) and two countries with a composite index of 13 (i.e., Guyana and Sudan). Iraq's index shows that it was lagged behind all other countries in the human, environmental, and economic dimensions. Notably, Iraq's composite index was higher than that of Yemen, despite the fact that the former obtained higher scores in the human and economic dimensions.

4.2.2. Temporal changes of sustainability from 2006 to 2016

To examine temporal changes in sustainability, we calculated the composite indices of each country (or region) using the SSI datasets for all available years—i.e., every two years from 2006 to 2016. Although the composite index had different value ranges (1–13 for the years of 2006, 2008, and 2012; 1–15 for 2010, 2014, and 2016), the world as a whole showed considerable progress in sustainability indicators over these 10 years. As shown in Fig. 5, the number (i.e., proportion) of countries (or regions) with a composite index of 1 displayed an upward

Table 7
Countries (or regions) that obtained high composite indices in 2016 and their detailed performance in terms of human, environmental, and economic dimensions.

Continent	Country (region)	Composite index	Human	Environmental	Economic
Asia	Iraq	15	12	7	7
Asia	Lebanon	14	10	7	6
Africa	Mauritania	14	15	5	8
Asia	Yemen	14	15	5	8
South America	Guyana	13	9	6	6
Africa	Sudan	13	15	5	7

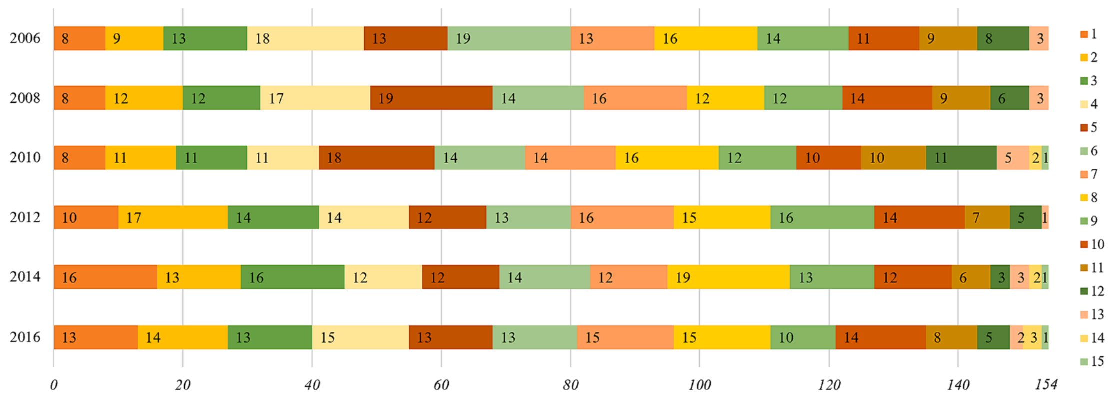

Fig. 5. Value distribution of the composite indices of each country (or region) by year.

trend from 8 in 2006 to 13 in 2016. This trend held for the total number of countries (or regions) with low values of the composite index. For example, the total number of countries (or regions) with a composite index of 1, 2, 3, or 4 increased from 48 in 2006 to 55 in 2016.

The next step was identifying the countries (or regions) that evinced continuously improved or weakened sustainability. To this end, four differences (Δ_1, Δ_2, Δ_3, and Δ_4) were calculated in the composite index of each country (or region) between the following four pairs of years (index of a more recent year minus that of a less recent year): 2006 and 2008, 2008 and 2012, 2010 and 2014, and 2014 and 2016, respectively. These four pairs were constructed according to two rules: contiguity and sharing the same composite index value range. If all the Δ_1, Δ_2, Δ_3, and Δ_4 values of a country (or region) were not smaller than zero and at least one of them was greater than 0, then the country (or region) was determined to have a continuous improvement of sustainability. However, if all of the Δ_1, Δ_2, Δ_3, and Δ_4 were not greater than 0 and at least one of them was smaller than 0, then the country (or region) was considered to be displaying weakened sustainability.

This method revealed that approximately 16% of countries (or regions) had continuously improved sustainability. Among these, Central African Republic had a composite index of 11 in 2006, which placed it among the worst 15% countries (or regions) around the world. However, the index continuously decreased from 2006 to 2016, reaching a value of 3 in 2016, thereby putting the Central African Republic among the top 20% countries (or regions) of that year. The main reason for this improvement is as follows. In the context of global resource consumption and environmental degradation, 42% countries (or regions) experienced a reduction in the Pareto score of the environmental dimension. In contrast, the Central African Republic had a stable performance in that dimension, making its advantage more evident. Moreover, Central African Republic made progress in the other two dimensions, namely the human dimension (from 18.0 in 2006 to 16.0 in 2016) and the economic dimension (from 7.0 in 2006 to 6.0 in 2016).

Only eight countries (or regions) experienced continuously weakened sustainability between 2006 and 2016. Among these, Vietnam's composite index increased from 2 in 2006 (ranked among the top 15% countries or regions) to 10 in 2016 (ranked among the bottom 30%). This decline was either due to simultaneous deterioration of its performance in all three dimensions or less progress in these three dimensions than the rest of the world.

5. Conclusions and discussion

The weighting of different indicators is among the core functions in evaluating the sustainable development of a region or the whole world. It is both an inevitable and critical step in almost all existing methods for constructing a composite index of sustainability based on a selection of indicators, along with pre-processing and aggregation. However, weighting is also the most controversial step, no matter whether it is carried out subjectively or objectively. To address this problem, much effort has been made toward developing more sophisticated, "objective" methods for weighting. This paper has proposed a novel, alternative direction of constructing composite indices by aggregating indicators without using their weights (i.e., indicators cannot be compared in terms of importance).

Toward this end, a weightless strategy was proposed for constructing composite indices of sustainability based on the mathematical optimization concept of Pareto fronts. We determined all possible implementations of the strategy, which formed four categories, namely, straightforward/hierarchical implementations with/without avoiding the accuracy issues of original indicators. To identify the most effective strategy, 24 typical implementations were conducted by covering all four categories and setting different parameters. Comparative experiments were then performed using two proposed measures of effectiveness, revealing the most effective strategy to be the hierarchical implementation using the GVF-based (GVF = 0.80) natural breaks to avoid accuracy issues. The strategy and this most effective implementation constitute our formal approach, which was applied to the world's sustainability by revising the well-known SSI to demonstrate its usefulness. If users are adopting the proposed weightless strategy, we also advise utilizing this implementation because the potential for accuracy issues to arise is prevalent in the majority of cases. Nonetheless, in instances where the outcomes lack distinctiveness, users may also refer to our tests to ascertain a more efficient implementation.

We conclude that the proposed weightless strategy and the tested implementation of this study can resolve controversies over weighting as well as aggregation. In fact, objections to aggregation are invariably linked to weighting. As noted by the two most important contributors to the SSI (Van de Kerk and Manuel, 2008, p. 233), "One of the objections to aggregation is that it can be compared to adding apples and oranges. However, if one accepts the definition of sustainability that has been used for the SSI, all 5 categories and 22 indicators are essential for assessing a country's sustainability—no matter whether they are apples or oranges." In line with this view, the study has removed the weighting reasons for objecting aggregation, and future efforts can be focused on improving aggregation itself.

As the first attempt to construct a weightless index, the proposed approach provides a novel paradigm of constructing composite indices of sustainability. Although the SSI was used as a case study, the proposed approach for constructing a composite index is universally applicable. All indicators of SSI are ratios, but it is important to note that the proposed approach also applies to interval (e.g., temperatures in Fahrenheit scale) and ordinal indicators (e.g., high, medium, and low). Another advantage of the proposed approach is its efficiency and robustness. The approach is efficient because its core calculation consists entirely of the determination of Pareto fronts, which can be easily performed with increasing numbers of indicators and assessment units. In contrast, existing approaches (e.g., analytic hierarchy process) usually have limitations on such numbers.

Future research is recommended in the following areas. The first is to explore the utility of crowdedness measures in determining the Pareto scores of assessment units. In this study, only domination was considered in the determination. The second is to develop methods for quantifying the effects of single indicators on the composite index calculated using the proposed approach. Such methods will further facilitate decision making for improving regional sustainability against the composite index.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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